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EXPERIMENTAL AND THEORETICAL IMPEDANCES AND ADMITTANCES OF CENTER-DRIVEN ANTENNAS

By

Phyllis A. Kennedy
and
Ronold King

April 1, 1953

Technical Report No. 155

Cruft Laboratory
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Cambridge, Massachusetts

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TR155

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Abstract

Recent experimental and theoretical results pertaining to the impedance and admittance of center-driven antennas have been consolidated into one report. A complete set of tables and curves with instructions for their use follows a general discussion of the problem. The essentials of the King-Middleton Second-Order Theory and the final equation for the second-order impedance are presented along with necessary spacing and end-effect corrections which must be considered. Experimental results from various sources have been compared with theory. In particular Hartig's experimental results on the effect of circular apertures in a horizontal ground plane on the impedance of a half-dipole have been reevaluated and found to agree well with theory. Special related topics such as the electrically short antenna and the receiving antenna have also been discussed.

TR155

Experimental and Theoretical Impedances and Admittances of Center-Driven Antennas

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Cruft Laboratory, Harvard University

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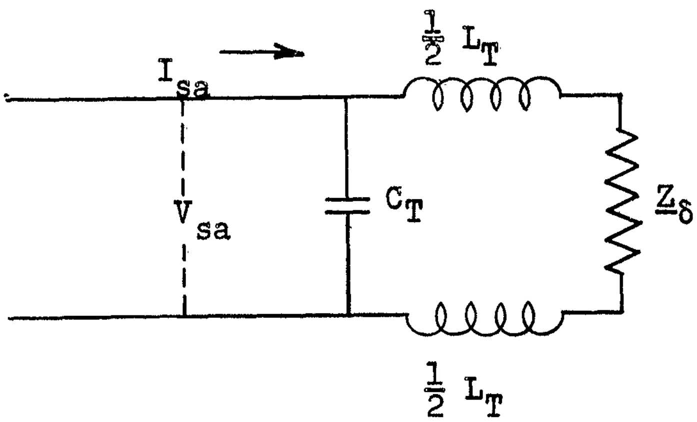
I

Theoretical Discussion of King-Middleton Second-Order Impedance

One of the most important types of transmitting antennas is the symmetrical, center-driven, straight, cylindrical conductor of small radius. While the antenna itself does not satisfy near-zone conditions, it must be driven either by a generator or transmission line which lies in the near zone, and therefore the two input terminals of the antenna may be considered in near-zone terms. From this point of view the antenna represents an impedance that may be defined as the ratio of applied voltage to input current.

The apparent or measurable terminal impedance $\underline{Z}_{sa}$ may be approximated by the impedance of a network combining an ideal impedance $\underline{Z}_{\delta}$ of the load with a lumped network consisting of a series inductance L_T and a parallel capacitance C_T which takes account of end-effect and coupling between the feeding line and the load. While these elements do not exist individually, they may be combined in the proper terminal network to permit calculation of a measurable transmission-line impedance. The evaluation of L_T and C_T for several types of transmission line is found in section VIII.



The load impedance Z_δ, which implies a finite separation 2δ of the terminals of the antenna, may be represented by an idealized impedance Z_0 with $\delta = 0$ plus a correction factor. This correction factor has been determined theoretically by expanding the admittance Y_δ in powers of $\beta_0\delta$ to obtain $Y_\delta - Y_0$ from which the correction $Z_\delta - Z_0$ may be computed. Curves of this correction factor for several values of Ω and as a function of $\beta_0\delta$ are in a later section on end effects. Thus, the determination of the apparent impedance Z_{sa} involves the theoretical evaluation of the elements L_T and C_T of the lumped network, the correction $Z_\delta - Z_0$, and Z_0. The following theoretical discussion is concerned with the impedance Z_0.

The second-order King-Middleton impedance can be calculated from the equation:*

$$(Z_0)_2 = \frac{1}{(Y_0)_2} = -j60\,\psi_{K1}\,\frac{\cos\beta_0 h + (D_1)_1 A_{1H}/\psi_{K1} + A_{2H}/\psi_{K1}^2}{(D_1)_2 \sin\beta_0 h + (D_2)_2 B_{1H}/\psi_{K1} + B_{2H}/\psi_{K1}^2}$$

(The order of solution is determined by the number of substitutions made in the method of successive approximations used in deriving the general formula for current.) The functions A and B of Eq. (1) may be defined as follows:

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*For a derivation of this formula, see R. King, Theory of Linear Antennas, Chap. II, to be published by the Harvard University Press; or R. King and D. Middleton, "The Cylindrical Antenna; Current and Impedance," Quart. Appl. Math. 3, 302 - 335, (January 1946).

$$\left.\begin{aligned} \underline{\underline{A}}_{1H} &= \alpha_1^{I} + j\alpha_1^{II} \\ \underline{\underline{B}}_{1H} &= \beta_1^{I} + j\beta_1^{II} \end{aligned}\right\} \quad (2)$$

$$\left.\begin{aligned} \underline{\underline{A}}_{2H} &= \alpha_2^{I} + j\alpha_2^{II} \\ \underline{\underline{B}}_{2H} &= \beta_2^{I} + j\beta_2^{II} \end{aligned}\right\} \quad (3)$$

where the first-order functions $\underline{\underline{\alpha}}_1$ and $\underline{\underline{\beta}}_1$ and the second-order functions $\underline{\underline{\alpha}}_2$ and $\underline{\underline{\beta}}_2$ are in the table I. The D-factors may also be defined,

$$\left.\begin{aligned} (D_1)_1 &= 1 + (1 - \frac{\Omega}{\Psi_{K1}}) \\ (D_1)_2 &= 1 + (1 - \frac{\Omega}{\Psi_{K1}}) + (1 - \frac{\Omega}{\Psi_{K1}})^2 \\ (D_2)_2 &= 1 + 2\,(1 - \frac{\Omega}{\Psi_{K1}}) \end{aligned}\right\} \quad (4)$$

where $\Omega = 2\,\ell n \frac{2h}{a}$, h = half-length of the antenna, and a = radius. The expansion parameter, Ψ_{K1}, may be considered a constant for a given value of Ω and a given half-length h of antenna as shown in Fig. 1. A more accurate determination of Ψ_{K1} for short antennas is discussed in section IV.

Numerical values of the King-Middleton second-order impedances and admittances as functions of $\beta_0 h$ have been tabulated for Ω = 7, 8, 9, 10, 11, 12.5, 15 and 20, and are given in Tables II-IX. For each value of Ω, $\frac{h}{a}$ is kept constant, thereby establishing the frequency as the variable in the term $\beta_0 h = \frac{2\pi}{\lambda} h$. In Figs. 2 through 4 the complex values of $(\underline{\underline{Z}}_0)_2$ have been plotted as functions of frequency with Ω as parameter. Corresponding curves of admittance are in Fig. 5. For convenience in extrapolating second-order impedances for intermediate values of Ω, resistance and

reactance have been plotted in Figs. 6 and 7 as functions of Ω with $\beta_0 h$ as parameter. In Figs. 8 and 9, R_0 and $|X_0|$ are plotted on a logarithmic scale as functions of $\beta_0 h$, and a linear plot of X_0 in the region near 0 is shown in Fig. 10. Similar curves for G_0 and $|B_0|$ are given in Figs. 11, 12, and 13. Theoretical values of impedance for $\Omega \geqq 10$ are quite accurate; however, as Ω is reduced from 10 to 7, the accuracy of the analysis becomes less good owing to the fact that the implied restrictions on the radius, $a << h$, $\beta_0 a = 2\pi a/\lambda_0 << 1$ are violated. Curves for which these conditions are not satisfied are drawn in broken line.

It is convenient theoretically to tabulate and plot impedance and admittance as functions of $\beta_0 h = \frac{2\pi h}{\lambda_0}$ with $\Omega = 2 \ln \frac{2h}{a}$ as parameter, and this procedure is also advantageous from an engineering standpoint where the length of an antenna is usually fixed and the frequency variable. However, laboratory measurements can be made more accurately and conveniently by maintaining a constant frequency and a fixed apparatus, in which only the actual half-length, h, of the antenna is varied. In this case a/λ_0, rather than Ω, is the constant parameter. Tabulated values of $\underline{Z}_0$ as a function of $\beta_0 h$ for eight values of d/λ_0 ($d=2a$) are in Table X, and corresponding curves are plotted in Figs. 14 and 15. Resistance and reactance as functions of a/λ_0 with $\beta_0 h$ as parameter are in Figs. 16 and 17. In Table XI are listed values of Ω for the given antenna diameters at intervals of $\beta_0 h$.

II. Terminal Functions

A cylindrical antenna usually is driven from a transmission line. Its property as a load impedance terminating a line of characteristic impedance $\underline{Z}_c$ may be expressed conveniently by the terminal functions ρ and Φ. The complex terminal function is defined as

$$\underline{\theta} = \rho + j\,\Phi = \coth^{-1} \underline{z}_1 \qquad (5)$$

where the normalized impedance of the load is

$$\underline{Z}_1 = r_1 + jx_1 = \underline{Z}/\underline{Z}_c \quad (\underline{Z}_c \doteq R_c \text{ for a low-loss line}). \tag{6}$$

The functions ρ and Φ characterize, respectively, the over-all attenuation of the load and its over-all phase shift. Figures 18 and 19 contain curves of ρ and Φ, for the second-order impedance of an antenna terminating a low-loss line, as functions of $\beta_o h$. Values of ρ and Φ have been calculated and plotted for antennas of $\Omega = 10$ and $\Omega = 20$ for several values of R_c. For each case, values of $(R_o)_{res}$ and $(R_o)_{antires}$ are given as well as the $\beta_o h$ values for which these conditions occur. It is evident that the function ρ will be infinite, and the function Φ discontinuous by $\pi/2$ at resonance or antiresonance, if R_c is equal to the resonant or antiresonant resistance.

III. Resonance and Antiresonance

Convenient reference points in the impedance or admittance of an antenna are those for which $X_o = 0$. These are known as resonant and antiresonant values.

Input resonance for thin antennas is defined as the condition for which X_o vanishes when $\beta_o h$ is near $\frac{n\pi}{2}$ (thin antennas) and n is odd. The resonant resistance is very nearly a minimum <u>except</u> when n = 1.

Input antiresonance is defined as the condition for which X_o vanishes when $\beta_o h$ is near $\frac{n\pi}{2}$ (thin antennas) and n is even. For antiresonance the resistance is very nearly a maximum. Additional values of $\underline{Z}_o$ near antiresonance are given in Table XII to facilitate the determination of the maximum impedance as well as the $\beta_o h$ value for which it occurs.

The $\beta_o h$ value at which resonance occurs is slightly smaller than $\pi/2$; and it can be seen from Fig. 20 where $\frac{n\pi}{2} - \beta_o h$ (n odd) is plotted as a function of Ω, that the resonant length of the antenna approaches $\frac{n\pi}{2}$(n odd) as Ω increases. Similarly, anti-

resonant $\beta_o h$ values are less than integral values of π, and curves for $\frac{n\pi}{2} - \beta_o h_{antires}$ (n even) are also plotted as functions of Ω.

The ratio of $|X_{min}/X_{max}|$, which is always greater than unity, is an important quantity in describing the behavior of the reactance. It can be seen in Fig. 9 that each capacitive lobe of the reactance curve is larger than the preceding inductive one, and that the reactance becomes increasingly capacitive for longer or thicker antennas.

Table XIII contains critical values near resonance and anti-resonance which have been computed or obtained graphically. Many of these quantities have also been plotted in Figs. 21 and 22 as functions of Ω.

Third-order conductances have been calculated recently,[1] for the particular case of $\beta_o h = \pi/2$, and these are given in Table XIV for comparison with second-order values. Since second-order reactances are known to be quite accurate at $\beta_o h = \pi/2$, these values may be used along with the new third-order conductances to calculate corresponding third-order resistance values. These are also found in Table XIV. It is evident from the results that the third-order conductances are lower and the third-order resistances higher than corresponding second-order terms. The percentage differences have been calculated and are given in the table.

IV. The Electrically Short Antenna

A center-driven cylindrical antenna is electrically short by definition if the condition

$$\beta_o h = \frac{2\pi h}{\lambda_o} < 1$$

is satisfied. For more quantitatively accurate solutions the condition $\beta_o h \leqq 0.5$ is necessary, but for many cases $\beta_o h \leqq 1$ is sufficiently restrictive.

Owing to the fact that the resistance of the short antenna is extremely small compared with the reactance, a modified procedure[2] for solving the integral equation by iteration is required if the resistance, as well as the reactance, is to be determined accurately.

This involves the use of a complex distribution of current as trial function in the integral equation and an expansion parameter defined as the ratio of the vector-potential difference (referred to the end of the antenna) to current rather than the ratio of the vector potential to current. This new parameter is more nearly constant over the short antenna than is the old. Numerical values of resistance and reactance have been calculated for $\Omega = 10$, using the improved procedure, and plotted in Fig. 23 as a function of $\beta_0 h$. For comparison the second-order King-Middleton impedance is plotted for $\Omega = 10$, and it is found that while corresponding reactance points overlap in a range for which both formulas are a good approximation, the King-Middleton second-order resistances are less accurate than the values from the new formula. Owing to the smallness of R_0 compared with X_0 and its consequent relative unimportance in the iteration, the second-order King-Middleton resistance is no more accurate than the zeroth-order resistance for $\Omega = \infty$

$$[R_0]_0 = 20\beta_0^2 h^2 (1 + \tfrac{2}{15}\beta_0^2 h^2) \tag{7}$$

which is obtained for a triangular distribution of current on the antenna.

There is no quick and accurate method of obtaining resistances of electrically short antennas, but a similar procedure may be followed to that by which numerical values for $\Omega = 10$ were obtained. For antennas with Ω between 10 and ∞, R_0 lies between $18.3\ \beta_0^2 h^2$ and $20\ \beta_0^2 h^2$ ohms.[2] A good approximation of the more accurate reactance formula for variable Ω is

$$X_0 = -\frac{60\Psi_{Dt}}{\beta_0 h}, \tag{8}$$

where

$$\Psi_{Dt} = \Omega - 2 - \ln 4 \tag{9}$$

Numerical values for Ω = 12.5, 15, and 20 have been calculated using this formula and interpolated curves drawn in Fig. 23 combining the resulting set of points and King-Middleton second-order values for reactance. Tabulated values of impedance as plotted in Fig. 23 can be found in Table XV.

V. A New Presentation of Hartig's Experimental Results[3]

It was the purpose of Hartig's research to investigate the effect of the circular aperture, or driving gap, at the base of a vertical half-dipole antenna when driven from a coxial line through a horizontal ground plane, on the impedance characteristic of that antenna. The data obtained include measurements for four different diameters of antenna and for each diameter, as many as five different diameters of the coaxial shield.

Since the publication of Hartig's report, his data have been reevaluated, and a better comparison between experiment and theory obtained. In the region of antiresonance where the resistance reaches a maximum, the data have been replotted to give more consistent curves (Figs. 24, 25, 26 and 27). In order to compare Hartig's results with the King-Middleton second-order theory which assumes $b/a = 1$, it is necessary to extrapolate any data of interest back to an experimentally unavailable $b/a = 1$ value. This was done in particular for the antiresonant region.

Values of maximum resistance were taken from the curves of Figs. 24 through 27 and new curves plotted in Fig. 28 for each thickness of antenna with maximum resistance as a function of b/a. When extrapolated to $b/a = 1$, the new values of maximum resistance are slightly lower than those predicted by the King-Middleton second-order theory. The location of R_{max} and antiresonance in terms of $\beta_o h$ are of considerable interest since antiresonance occurs near a point of maximum resistance. Curves of these critical values for four thicknesses of antenna have been plotted in Fig. 28 as functions of b/a, and then extrapolated back to $b/a = 1$. In addition, corresponding points from the King-Middleton

second-order theory have been plotted on the appropriate curves. It can be seen from Fig. 28 that these theoretical points are in each case slightly higher, or correspond to slightly longer antennas, than the extrapolated experimental values, and that closeness of agreement between theory and experiment decreases as the antenna becomes thicker. This reaffirms the conclusion that the theory is more accurate for thin antennas. It will also be noticed that the values of $\beta_0 h$ for which R_{max} occurs are higher in each case than those for antiresonance.

From the given values of b/a and the corresponding values of $\beta_0 h$, Ω's for antiresonance and maximum resistance were calculated using the formula $\Omega = 2\ell n\frac{2h}{a}$. Curves of Ω (anti-resonant) and Ω (R_{max}) also were plotted as functions of b/a. As might be expected, these sets of curves differ slightly for each value of a/λ_0 and their differences increase with the thickness of antenna.

In Figs. 29 and 30 where the measured resistance is plotted over a wide range of $\beta_0 h$, the region of importance is that near resonance. Figures 29 through 32 show enlarged plots of this region for four thicknesses of antenna. Third-order-resistance points for $\beta_0 h = \pi/2$ [1] (see Table XIV) are shown on the appropriate curves. They agree very well with the experimental results, and the agreement is best for thin antennas.

The admittance near resonance has been considered for a representative case in Fig. 33--that of the moderately thick antenna. As described previously, Hartig's data were replotted and extrapolated back to b/a = 1 in order to obtain a new curve. In addition, a theoretical b/a = 1 curve has been plotted from the data given in Table X for the antenna of $a/\lambda_0 = 9.52 \times 10^{-3}$, along with a third-order point for $\beta_0 h = \pi/2$. This point lies extremely close to the experimental curve.

A tabulation of Hartig's measured impedances as used in this presentation is given in Table XVI.

VI. The Impedance of the Receiving Antenna

Since the impedance of a receiving antenna in the far zone of a transmitter is by definition the same as the impedance of the same antenna when driven, it may be determined from measurements on a slotted line loading the antenna. It is assumed that the receiving antenna is in the far zone of the transmitter. The distributions of charge and current in the receiving antenna, therefore, have a negligible effect on current in the transmitter, and the electromagnetic field may be defined in terms of current distribution already obtained for the transmitting antenna.

Measurements have been made by Wilson and Hartig and reported by Hartig, Morita, King and Wilson,[4] comparing experimental impedances measured on the same antenna when used successively for transmitting and receiving. The experimental results are in Figs. 34 through 36, and may be compared with the theoretical King-Middleton second-order values calculated previously for the transmitting antenna (see Tables II through IX). In Fig. 34 the impedance is plotted over a wide range of $\beta_0 h$ with corresponding Ω-values on a separate scale. The impedances near resonance and antiresonance are plotted respectively in Figs. 35 and 36. It is evident that the two sets of experimental values are in excellent agreement with each other and with theory.

VII. Comparison of Various Experimental Results with Theory

Results obtained from various sources have been compiled and plotted in Figs. 37 through 39. Certain critical values such as the points of maximum resistance, the conductance near resonance, and the resistance at resonance are of particular interest. Figure 37 presents the resistance maxima for the cylindrical antenna as functions of thickness of the antenna. Theoretical curves based on the Hallén second-order, King-Middleton second-order, and Schelkunoff first-order formulas have been presented for purposes of comparison. Data from Hallén and

and King-Middleton are available for both the first and second maximum points, while the Schelkunoff data are available only for the first maximum. Experimental points have been plotted on the curves and are found in general to be in better agreement with the King-Middleton theoretical curves than with either of the others. Consideration of the second maximum reveals that the only experimental data available, those of Wilson, agree quite well with the King-Middleton curve.

When any admittance characteristic is plotted as a function of $\beta_0 h$ in the region near resonance, any of three separate critical values may be of importance. For example, if one wishes to consider the conductance, the resonant value, the maximum value, and the value at $\beta_0 h = \pi/2$ are particularly interesting. All three of these occur at different values of $\beta_0 h$. $[G_0]_{resonant}$ will usually be found at a value of $\beta_0 h$ which is less than $\pi/2$ and also less than that for the other two critical quantities. The value of $\beta_0 h$ at which G_0 reaches a maximum is slightly greater than that for $[G_0]_{res}$ but still less than $\pi/2$. The conductance at $\beta_0 h = \pi/2$ is also of interest, and its magnitude is smaller than either of the other conductances mentioned. This is a result of the fact that the slope of the conductance curve increases rapidly after the maximum is reached. An estimate of the relative positions of these critical values may be obtained from Figs. 10 and 11 where King-Middleton second-order data are plotted.

In Fig. 38 several theoretical curves of maximum conductance and the King-Middleton curve for conductance at resonance are compared with various experimental points. It will be noted that the King-Middleton curve for $[G_0]_{res}$ falls below that for $[G_0]_{max}$. Correspondingly, $[G_0]_{res}$ curves for Schelkunoff and Hallén would fall below their respective $[G_0]_{max}$ curves, if such data were available. Presumably, a third-order $[G_0]_{res}$ curve would be an even more accurate theoretical result. Such a curve has been obtained using a percentage difference method. The difference between second-order and third-order results for

the $[G_{0} \cdot \pi/2$ case (see Table XIV) is found and transformed into a percentage of the second-order conductance. This percentage is applied to the second-order conductance at resonance to obtain an extrapolated third-order resonant conductance. This curve can be seen to agree better with the experimental data although the contour of the new curve in the theoretically less accurate region of thicker antennas is somewhat different.

Curves of resistance at resonance are in Fig. 39. The Schelkunoff curve is much lower than the King-Middleton second- and third-order curves and is in poorer agreement with experiment. The King-Middleton third-order curve was determined by the percentage difference method described previously, and this extrapolated curve falls above the second-order curve in the manner expected. The experimental points are those of D. D. King and E. O. Hartig. Hartig's three points were taken from Figs. 29, 30 and 31 and obtained more accurately by plotting the critical region on a very large scale and extrapolating the resonant resistance points. The points included in circles indicate the resonant resistance for the available line spacing which most nearly approximates the theoretical value of b/a = 1. The range designated by the spread on the curve is the range included by the larger ratios of b/a.

VIII. Coupling-and End-Effects

A rigorous determination of the impedance of a symmetrical center-driven antenna terminating a two-wire or coaxial line requires the solution of simultaneous equations in the distributions of current in both the antenna and the transmission line. The true transmission-line impedance can be defined only for points outside a terminal zone while the ideal antenna impedance of an isolated antenna independent of the driving transmission line cannot be measured. This is a consequence of transmission-line end-effects and of coupling between the antenna and transmission line. The end-effect due to the spacing 2δ (or b) at the terminals of the antenna requires a correction

factor $\underline{Z}_\delta - \underline{Z}_o$ described in section I. Curves of resistive and reactive corrections as functions of $\beta_o\delta$ and with $\beta_o h$ as parameter are given in Figs. 40, 41 and 42 for several values of Ω . These effects are significant over distances along antenna and line that are comparable to small multiples of the separation b. The condition $\beta_o b << 1$ effectively confines the coupling to the near zone, and since conditions of symmetry and perpendicularity for the symmetrical center-driven antenna eliminate inductive coupling, the coupling between antenna and line may be represented by a lumped capacitance in parallel with the antenna at its junction with the line. The equivalent circuit for the terminal zone of an antenna as end load on a two-wire line is shown in Fig. 43, where the subscript Te indicates an end-loaded line. The lumped series inductance L_{Te} compensates for the use of a constant inductance per unit length in the terminal zone. The lumped parallel capacitance C_{Te} includes a correction for the assumed constant capacitance per unit length of line in the terminal zone as well as for the effect of coupling between the line and the termination. For the circuits of Fig. 43

$$L_{Te} = -\frac{(b - a)}{2\pi v_o} , \qquad (10)$$

where

$$v_o = \frac{1}{4\pi} \times 10^7 \text{ m/henry} , \quad \text{and}$$

$$-C_{Te}/bc_o$$

is defined and plotted in Fig. 44. The following notation is used:

- $b = 2\delta$ is the full-line spacing,
- a is the radius of the line,
- d is the length of the terminal zone,
- w is the distance from the point on the line under consideration to the antenna,
- c_o is the uniform capacitance per unit length of line.

Figure 45(a) shows an antenna mounted at the center of a symmetrical line driven from both ends by identical generators where each line may be considered to be terminated at the center in a combined impedance $2Z_\delta$. The values of the lumped reactances must now be redefined. L_{Tc}, where the subscript Tc represents the center-loaded line, is found to be just double the value for the end-loaded line.

$$L_{Tc} = - \frac{b-a}{\pi v_o} \quad . \tag{11}$$

This value of lumped inductance must be connected in series with each half of the symmetrical line. The lumped capacitance is defined and plotted in Fig. 46.

In Fig. 45(b) the equivalent circuit which replaces the antenna as a center-load is given, and for which there are alternative equivalent circuits in Figs. 45(c) and 45(d). In Fig. 45(c) each side of the circuit may be treated separately since no current crosses the central axis of symmetry. In Fig. 45(d) a circuit for the image line is represented. Experimentally the theorem of images facilitates the study of center-driven antennas for applications of center-loaded or end-loaded lines. In Fig. 45(d) the horizontal dotted line represents the plane $z = 0$ which is a plane of symmetry for the circuit. On this plane the tangential component of the electric field is zero everywhere, and a perfectly conducting sheet may be placed at $z = 0$, isolating both halves of the circuit. One half of the line may then be removed, leaving the distributions of current and charge and the electromagnetic field in the remaining half of the line unchanged.

An antenna with a stub support may be used instead of the dielectric-supported end-load antenna previously mentioned. The stub consists of a high-impedance, antiresonant section of line, the adjustment of which is quite important; the position of antiresonance on the line should be determined accurately, and then the antenna mounted at this point. With the stub constructed in this way so that its impedance is very large compared with that

of the antenna, the current in the stub near the junction of line and antenna is very small, and inductance per unit length of the transmission line differs negligibly from that for the unsupported end-loaded line. Thus the value of $L_T = L_{Te}$ may be used for the lumped inductance in the equivalent circuit. When the voltage is a maximum at the terminals of the antenna, the distributions of voltage and charge along the transmission line and stub are essentially the same as for the symmetrical center-loaded line discussed earlier. Therefore C_T may be considered equal to C_{Tc} on each side of the antenna. The equivalent circuit for the stub-supported antenna is given in Fig. 47. The current in the stub is quite small and no inductance is required on that side of the antenna. Furthermore, since L_{Te} is small, the parallel capacitance may be combined as shown in Fig. 47(c).

It is possible to orient a particular antenna in such a way that there is negligible coupling to the line. In order to accomplish this, it is necessary that the antenna be perpendicular to the plane of all other conductors and in their plane of symmetry so that the vector and scalar potentials due to currents and charges on the transmission line vanish at all points along the antenna. By mounting the antenna in a plane perpendicular to the line with a high-impedance stub support adjusted to give a voltage maximum at the terminals of the antenna (see Fig. 48), it is possible to compensate for virtually all coupling-and end-effects except for small values associated with the short connections between antenna and line, so that $\underline{Z}_{sa} \doteq \underline{Z}_{\delta}$.

An approximate solution for the end-effect correction which must be considered when a dipole is driven over an image plane by a coaxial line of finite line spacing is in Hartig's report.[3] Again all significant effects are confined to regions near the end of the line and may be represented by lumped circuit elements. Since the inductance per unit length does not change near the end of the line, a corrective inductance is not required. The corrective capacitance is defined as

$$-\frac{C_T}{bc_o} = \int_0^d \frac{1}{b} \frac{\psi}{2\,\ell n \frac{b}{a} + \psi} \, dw \quad , \qquad (12)$$

where b = radius of the outer conductor of the coaxial line,

a = radius of the inner conductor,

and

$$\psi = \frac{1}{4}\left[\sinh^{-1}\frac{(1-\frac{a}{b})}{(\frac{w}{b})} + \ell n(1 + \frac{w^2}{b^2}) - \ell n(\frac{a^2}{b^2} + \frac{w^2}{b^2})\right] \qquad (13)$$

Approximate corrective capacitances obtained from this integral have been evaluated numerically and plotted as a function of b/a in Fig. 49. In addition, Hartig has obtained corresponding experimental points for his thin and moderately thin antennas. Comparison of these experimental and theoretical curves in Fig. 49 indicates that, except for small line spacings, the experimental points fall on a smooth curve which lies slightly above that obtained theoretically.

Many of the procedures for mounting and supporting antennas and the corresponding theoretical corrections which have been described in this section have been applied experimentally. In a report on the theory and summary of measurements, R. King[5] has presented several pertinent curves comparing experimental results and theoretical data to which an end-effect correction has been applied. Impedance comparisons for the case of the high-impedance stub-supported end-load are given in Figs. 24(a) and 24(b) of King's paper[5] and in Figs. V-12 and V-13 of the original discussion by K. Tomiyasu.[6] Curves for the impedance of an antenna connected as center-load in the plane of a transmission line, which is driven at both ends, are available in Figs. 25(a), (b), (c) and 26 of reference 5 and also in Figs. 15, 20, 21, 22 and 23 of a report by Conley[7] in which he describes open-wire measurements. Tomiyasu also discusses the stub-supported antenna mounted perpendicular to the plane of the line, a case for which no correction is needed. His results are compared with King-Middleton second-order impedance values in Fig. 27 of reference 5 and also

in Fig. V-8 of reference 6. Angelakos,[8] using essentially the equipment of Conley, has plotted the impedance variation for the theoretical case, the styrofoam-supported end-load, and the stub-supported end-load. (See Fig. VII-6 of reference 8.)

IX Summary of Procedure

When the frequency of operation, the thickness and length of antenna, and the particular orientation of antenna and line are known, it is possible to predict $\underline{Z}_{sa}$, the apparent impedance of the antenna measured along the transmission line.

1. Determine the theoretical impedance $\underline{Z}_0$ from the King-Middleton second-order values in Tables II - XII.

2. The load impedance $\underline{Z}_\delta$ is found by applying to $\underline{Z}_0$ a correction for the spacing 2δ which is obtained from Figs. 40, 41 and 42.

3. The equivalent circuit for the particular orientation of the antenna must be determined as in Figs. 43, 45, 47 and 48, and the equivalent lumped elements evaluated from the appropriate curves or the equation in section VIII.

4. The problem of determining $\underline{Z}_{sa}$ from $\underline{Z}_\delta$ is now simplified to the application of ordinary circuit theory.

References

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References

4. E. O. Hartig, T. Morita, R. King, and D. G. Wilson, "The Measurement of Antenna Impedance Using a Receiving Antenna," Cruft Laboratory Technical Report No. 94, December, 1949; Proc. I.R.E. 39, 1458-1460 (November, 1951).

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6. K. Tomiyasu, "Problems of Measurement on Two-Wire Lines with Application to Antenna Problems, "Cruft Laboratory Technical Report No. 48, June 15, 1948. "Antennas and Open-Wire Lines, Part II, Measurements on Two-Wire Lines," J. App. Phys. 20, 892-896 (1949).

7. P. Conley, "Impedance Measurements on Open-Wire Lines," Cruft Laboratory Technical Report. No. 35, March 18, 1948. "Antennas and Open-Wire Lines, Part III, Image-Line Measurements," J. App. Phys. 20, 1022-1026 (1949).

8. D. Angelakos, "Current and Charge Distributions on Antennas and Open-Wire Lines," Cruft Laboratory Technical Report No. 98, March 1, 1950. J. App. Phys. 20, 1022-1026 (1949).

TABLE I*

$\beta_0 h$	$\cos\beta_0 h$	$\underline{\alpha}_1 = \alpha_1^{I} + j\alpha_1^{II}$	$\underline{\alpha}_2 = \alpha_2^{I} + j\alpha_2^{II}$	$\sin\beta_0 h$	$\underline{\beta}_1 = \beta_1^{I} + j\beta_1^{II}$	$\underline{\beta}_2 = \beta_2^{I} + j\beta_2^{II}$
0,0	+1,000000	—0,0000 +0,0000j	—0,0000 +0,0000j	+0,000000	+0,0000 +0,0000j	+ 0,0000 + 0,0000j
0,1	+0,995004	—0,0100 +0,0006j	—0,0360 +0,0022j	+0,099833	+0,3374 +0,0000j	+ 1,3462 + 0,0002j
0,2	+0,980067	—0,0393 +0,0053j	—0,1415 +0,0171j	+0,198669	+0,6667 +0,0006j	+ 2,6426 + 0,0030j
0,3	+0,955336	—0,0864 +0,0175j	—0,3100 +0,0563j	+0,295520	+0,9802 +0,0028j	+ 3,8420 + 0,0192j
0,4	+0,921061	—0,1490 +0,0407j	—0,5311 +0,1285j	+0,389418	+1,2710 +0,0083j	+ 4,9014 + 0,0591j
0,5	+0,877583	—0,2234 +0,0773j	—0.7920 +0,2302j	+0,479426	+1,5326 +0,0197j	+ 5,7838 + 0,1309j
0,6	+0,825336	—0,3061 +0,1293j	—1,0784 +0,3901j	+0,564642	+1,7594 +0,0397j	+ 6,4590 + 0,2792j
0,7	+0,764842	—0,3925 +0,1974j	—1,3762 +0,5790j	+0,644218	+1,9475 +0,0713j	+ 6,9070 + 0,4946j
0.8	+0,696707	—0,4781 +0,2816j	—1,6723 +0,8004j	+0,717356	+2,0938 +0,1176j	+ 7,1141 + 0,8018j
0,9	+0,621610	—0,5583 +0,3812j	—1,9559 +1,0462j	+0,783327	+2,1960 +0,1811j	+ 7,0723 + 1,2134j
1,0	+0,540302	—0,6291 +0,4935j	—2,2191 +1,3069j	+0,841471	+2,2540 +0,2641j	+ 6,7825 + 1,7374j
1,1	+0,453596	—0,6866 +0,6157j	—2,4564 +1,5728j	+0,891207	+2,2682 +0,3681j	+ 6,2504 + 2,3770j
1,2	+0.362358	—0,7278 +0,7450j	—2,6654 +1,8344j	+0,932039	+2,2403 +0,4941j	+ 5,4859 + 3,1306j
1,3	+0,267499	—0,7504 +0,8778j	—2,8455 +2,0844j	+0,963558	+2,1725 +0.6123j	+ 4,5019 + 3,9915j
1,4	+0,169967	—0,7527 +1,0090j	—2,9977 +2,3171j	+0,985450	+2,0681 +0,8112j	+ 3,3135 + 4,9405j
1,5	+0,070737	—0,7345 +1,1351j	—3,1232 +2.5201j	+0,997495	+1.9308 +0,9996j	+ 1,9376 + 5,9908j
1,6	—0,029200	—0,6957 +1,2517j	—3,2220 +2,7192j	+0;999574	+1,7644 +1,2042j	+ 0,3921 + 7,0994j
1,7	—0,128844	—0,6377 +1,3550j	—3,2969 +2,8875j	+0,991665	+1,5731 +1,4216j	— 1,3036 + 8,2580j
1,8	—0,227202	—0,5619 +1,4419j	—3,3436 +3,0354j	+0,973848	+1,3601 +1,6477j	— 3,1291 + 9,4484j
1,9	—0,323290	—0,4708 +1,5097j	—3,3604 +3,1643j	+0,946300	+1,1301 +1,8781j	— 5,0620 +10,6524j
2,0	—0,416147	—0,3673 +1,5562j	—3,3431 +3,2752j	+0,909297	+0,8864 +2,1071j	— 7,0786 +11,8517j
2,1	—0,504846	—0,2541 +1,5805j	—3,2867 +3,3680j	+0,863209	+0,6317 +2,3304j	— 9,1530 +13,0282j
2,2	—0,588501	—0,1343 +1,5819j	—3,1862 +3,4413j	+0,808496	+0,3691 +2,5431j	—11,2576 +14,1641j
2,3	—0,666276	—0,0108 +1,5605j	—3,0369 +3,4022j	+0,745705	+0,1011 +2,7393j	—13,3631 +15,2414j
2,4	—0,737394	+0,1134 +1,5170j	—2,8350 +3,5166j	+0,675463	—0,1701 +2,9154j	—15,4390 +16,2421j
2,5	—0,801144	+0,2360 +1,4528j	—2,5787 +3,5091j	+0,598472	0,4425 +3,0676j	—17,4539 +17,1481j
2,6	—0,856889	+0,3552 +1,3695j	—2,2678 +3,4640j	+0,515501	—0,7142 +3,1931j	—19,3764 +17,9411j
2,7	—0,904072	+0,4687 +1,2691j	—1,9045 +3,3755j	+0,427380	—0,9833 +3,2883j	—21,1756 +18,6030j
2,8	—0,942222	+0,5756 +1,1534j	—1,4931 +3,2385j	+0,334988	—1,2479 +3,3520j	—22,8220 +19,1159j
2,9	—0,970958	+0,6750 +1,0247j	—1,0397 +3,0487j	+0,239249	—1,5065 +3,3828j	—24,2878 +19,4628j
3,0	—0,989992	+0,7662 +0,8851j	—0,5524 +2,8034j	+0,141120	—1,7560 +3,3799j	—25,5479 +19,6277j
3,1	—0,999135	+0,8487 +0,7362j	—0,0402 +2,5018j	+0,041581	—1,9946 +3,3422j	—26,5798 +19,5964j
3,2	—0,998295	+0,9225 +0,5800j	+0,4869 +2,1448j	—0,058374	—2,2200 +3,2732j	—27,3644 +19,3568j
3,3	—0,987480	+0,9876 +0,4181j	+1,0187 +1,7349j	—0,157746	—2,4202 +3,1705j	—27,8859 +18,8991j
3,4	—0,966798	+1,0435 +0,2515j	+1,5450 +1,2770j	—0,255541	—2,6196 +3,0361j	—28,1322 +18,2165j

*From King and Blake, Proc. I.R.E. 30, 337 (1942) and E. Hallén, Trans. Royal Inst. Tech, No. 13, p. 10 (1947)

TABLE I (CONT.)

$\beta_0 h$	$\cos \beta_0 h$	$\underline{\alpha}_1 = \alpha_1^I + j\alpha_1^{II}$	$\underline{\alpha}_2 = \alpha_2^I + j\alpha_2^{II}$	$\sin \beta_0 h$	$\underline{\beta}_1 = \beta_1^I + j\beta_1^{II}$	$\underline{\beta}_2 = \beta_2^I + j\beta_2^{II}$
3,5	—0,936457	+1,0905 +0,0816j	+2,0560 +0,7771j	—0,350783	—2,7884 +2,8713j	—28,0945 +17,3054j
3,6	—0,896758	+1,1280 —0,0903j	+2,5430 +0,2427j	—0,442520	—2,9325 +2,6772j	—27,7679 +16,1655j
3,7	—0,848100	+1,1556 —0,2634j	+2,9979 —0,3177j	—0,529836	—3,0495 +2,4552j	—27,1510 +14,7998j
3,8	—0,790968	+1,1731 —0,4364j	+3,4141 —0,8948j	—0,611858	—3,1375 +2,2070j	—26,2459 +13,2150j
3,9	—0,725932	+1,1795 —0,6082j	+3,7862 —1,4792j	—0,687766	—3,1945 +1,9344j	—25,0578 +11,4213j
4,0	—0,653644	+1,1743 —0,7772j	+4,1100 —2,0610j	—0,756802	—3,2188 +1,6393j	—23,5957 + 9,4320j
4,1	—0,574824	+1,1569 —0,9422j	+4,3828 —2,6312j	—0,818277	—3,2096 +1,3235j	—21,8709 + 7,2642j
4,2	—0,490261	+1,1265 —1,1017j	+4,6030 —3,1808j	—0,871576	—3,1662 +0,9889j	—19,8980 + 4,9376j
4,3	—0,400799	+1,0829 —1,2535j	+4,7700 —3,7021j	—0,916166	—3,0891 +0,6383j	—17,6939 + 2,4748j
4,4	—0,307333	+1,0253 —1,3960j	+4,8839 —4,1880j	—0,951602	—2,9790 +0,2743j	—15,2780 — 0,0991j
4,5	—0,210796	+0,9541 —1,5271j	+4,9459 —4,6330j	—0,977530	—2,8369 —0,1003j	—12,6714 — 2,7570j
4,6	—0,112153	+0,8695 —1,6448j	+4,9568 —5,0323j	—0,993691	—2,6647 —0,4823j	— 9,8976 — 5,4705j
4,7	—0,012389	+0,7719 —1,7467j	+4,9180 —5,3825j	—0,999923	—2,4645 —0,8681j	— 6,9814 — 8,2099j
4,8	+0,087499	+0,6625 —1,8316j	+4,8308 —5,6811j	—0,996165	—2,2390 —1,2535j	— 3,9490 —10,9449j
4,9	+0,186512	+0,5424 —1,8974j	+4,6959 —5,9263j	—0,982453	—1,9903 —1,6345j	— 0,8285 —13,6451j
5,0	+0,283662	+0,4129 —1,9426j	+4,5142 —6,1171j	—0,958924	—1,7214 —2,0072j	+ 2,3511 —16,2805j
5,1	+0,377978	+0,2763 —1,9665j	+4,2860 —6,2526j	—0,925815	—1,4356 —2,3666j	+ 5,5596 —18,8212j
5,2	+0,468517	+0,1345 —1,9683j	+4,0117 —6,3321j	—0,883455	—1,1361 —2,7086j	+ 8,7656 —21,2388j
5,3	+0,554374	—0,0108 —1,9475j	+3,6915 —6,3547j	—0,832267	—0,8252 —3,0290j	+11,9367 —23,5055j
5,4	+0,634693	—0,1572 —1,9044j	+3,3261 —6,3196j	—0,772764	—0,5062 —3,3234j	+15,0407 —25,5949j
5,5	+0,708670	—0,3026 —1,8394j	+2,9167 —6,2254j	—0,705540	—0,1824 —3,5880j	+18,0441 —27,4821j
5,6	+0,775566	—0,4448 —1,7536j	+2,4651 —6,0711j	—0,631267	+0,1437 —3,8194j	+20,9140 —29,1438j
5,7	+0,834713	—0,5822 —1,6480j	+1,9742 —5,8555j	—0,550686	+0,4696 —4,0142j	+23,6183 —30,5577j
5,8	+0,885520	—0,7131 —1,5241j	+1,4480 —5,5774j	—0,464602	+0,7922 —4,1701j	+26,1252 —31,7044j
5,9	+0,927478	—0,8359 —1,3873j	+0,8916 —5,2364j	—0,373877	+1,1087 —4,2854j	+28,4052 —32,5655j
6,0	+0,960170	—0,9496 —1,2287j	+0,3111 —4,8327j	—0,279415	+1,4166 —4,3583j	+30,4303 —33,1255j
6,1	+0,983268	—1,0528 —1,0608j	—0,2862 —4,3674j	—0,182163	+1,7134 —4,3874j	+32,1751 —33,3710j
6,2	+0,996542	—1,1450 —0,8821j	—0,8922 —3,8426j	—0,083089	+1,9963 —4,3725j	+33,6173 —33,2915j
6,3	+0,999859	—1,2256 —0,6946j	—1,4983 —3,2620j	+0,016814	+2,2623 —4,3145j	+34,7378 —32,8797j
6,4	+0,993185	—1,2910 —0,5002j	—2,0957 —2,6302j	+0,116549	+2,5087 —4,2133j	+35,5210 —32,1313j
6,5	+0,976588	—1,3497 —0,3005j	—2,6753 —1,9533j	+0,215120	+2,7331 —4,0697j	+35,9550 —31,0456j
6,6	+0,950233	—1,3926 —0,0976j	—3,2286 —1,2385j	+0,311541	+2,9325 —3,8852j	+36,0321 —29,6257j
6,7	+0,914383	—1,4224 +0,1067j	—3,7474 —0,4941j	+0,404850	+3,1054 —3,6621j	+35,7484 —27,8787j
6,8	+0,869397	—1,4387 +0,3111j	—4,2243 +0,2707j	+0,494113	+3,2485 —3,4022j	+35,1043 —25,8153j
6,9	+0,815725	—1,4412 +0,5135j	—4,6530 +1,0462j	+0,578440	+3,3602 —3,1072j	+34,1037 —23,4505j
7,0	+0,753902	—1,4301 +0,7123j	—5,0281 +1,8222j	+0,656987	+3,4392 —2,7801j	+32,7545 —20,8028j

Table II

King-Middleton Second-Order Impedances

$$\Omega = 2\ln\frac{2h}{a} = 7 \qquad h/a = 16.56$$

$\beta_o h$	Z_o(ohms)	$\lvert Z_o \rvert$	Y_o(mhos x 10^{-3})	$\lvert Y_o \rvert$ x 10^{-3}
0.5	4.985 -j375.6	375.6	0.035 +j 2.662	2.662
0.7	10.38 -j237.4	237.6	0.184 +j 4.204	4.208
0.9	18.66 -j150.6	151.8	0.810 +j 6.539	6.589
1.1	31.31 -j 85.40	90.96	3.785 +j10.32	10.99
1.2	39.84 -j 56.87	69.43	8.264 +j11.80	14.40
1.3	50.64 -j 30.05	58.88	14.61 +j 8.667	16.99
1.4	64.06 -j 4.582	64.23	15.53 +j 1.111	15.57
1.5	80.83 +j 19.70	83.20	11.68 -j 2.846	12.02
1.6	101.8 +j 42.51	110.3	8.365 -j 3.493	9.065
1.7	129.1 +j 63.01	143.7	6.255 -j 3.053	6.961
1.8	164.8 +j 78.42	182.5	4.947 -j 2.354	5.478
1.9	210.2 +j 83.60	226.2	4.108 -j 1.634	4.421
2.0	264.0 +j 69.99	273.1	3.539 -j 0.938	3.661
2.1	315.9 +j 27.76	317.2	3.140 -j 0.276	3.152
2.2	344.6 -j 44.82	347.5	2.853 +j 0.371	2.877
2.3	330.5 -j127.0	354.1	2.636 +j 1.013	2.824
2.4	279.0 -j187.6	336.2	2.469 +j 1.660	2.975
2.5	215.2 -j214.4	303.8	2.332 +j 2.323	3.291
2.6	158.2 -j215.2	267.1	2.218 +j 3.016	3.744
2.7	114.8 -j202.5	232.8	2.119 +j 3.738	4.296
2.8	83.36 -j184.6	202.6	2.032 +j 4.499	4.937
2.9	61.37 -j165.9	176.9	1.961 +j 5.302	5.653
3.0	46.08 -j148.1	155.1	1.915 +j 6.155	6.446
3.1	35.77 -j132.0	136.7	1.913 +j 7.059	7.314
3.2	28.95 -j117.8	121.3	1.968 +j 8.008	8.246
3.3	24.72 -j105.0	107.9	2.124 +j 9.021	9.268
3.4	22.43 -j 93.61	96.26	2.421 +j10.10	10.39
3.5	21.54 -j 83.35	86.09	2.906 +j11.25	11.62
3.6	21.82 -j 73.79	76.95	3.685 +j12.46	12.99
3.7	22.96 -j 64.61	68.57	4.883 +j13.74	14.58
3.8	24.87 -j 55.50	60.82	6.723 +j15.00	16.44
3.9	27.61 -j 46.21	53.83	9.527 +j15.94	18.57
4.0	31.20 -j 36.40	47.94	13.58 +j15.84	20.86

Table III

King-Middleton Second-Order Impedances

$\Omega = 2\ln \frac{2h}{a} = 8 \quad h/a = 27.30$

$\beta_o h$	Z_o(ohms)	$\lvert Z_o \rvert$	Y_o(mhos x 10^{-3})	$\lvert Y_o \rvert$ x 10^{-3}
0.5	4.984 -j484.5	484.6	0.021 + j 2.063	2.063
0.7	10.30 -j308.4	308.6	0.108 + j 3.239	3.240
0.9	18.49 -j198.2	199.0	0.467 + j 5.002	5.024
1.1	30.70 -j116.0	120.0	2.132 + j 8.056	8.333
1.2	38.94 -j 80.11	89.07	4.908 + j10.10	11.23
1.3	49.14 -j 46.43	67.60	10.75 + j10.16	14.79
1.4	61.83 -j 14.11	63.42	15.37 + j 3.508	15.76
1.5	77.59 +j 17.28	79.49	12.28 - j 2.735	12.58
1.6	97.41 +j 47.93	108.6	8.262 - j 4.065	9.208
1.7	123.0 +j 77.81	145.5	5.807 - j 3.674	6.872
1.8	157.1 +j105.6	189.3	4.384 - j 2.947	5.282
1.9	202.1 +j128.5	239.5	3.523 - j 2.240	4.175
2.0	261.1 +j140.7	296.6	2.968 - j 1.599	3.370
2.1	334.1 +j131.5	359.0	2.592 - j 1.020	2.785
2.2	411.6 +j 86.47	420.6	2.327 - j .489	2.378
2.3	469.4 -j 2.858	469.4	2.131 + j .013	2.131
2.4	474.3 -j118.5	488.9	1.984 + j .496	2.045
2.5	421.2 -j218.4	474.4	1.871 + j .970	2.107
2.6	338.9 -j274.3	436.0	1.783 + j 1.443	2.294
2.7	259.0 -j290.3	389.1	1.711 + j 1.917	2.569
2.8	194.5 -j282.1	342.6	1.657 + j 2.403	2.919
2.9	146.7 -j263.1	301.2	1.617 + j 2.899	3.320
3.0	112.6 -j240.6	265.6	1.595 + j 3.409	3.763
3.1	88.63 -j218.0	235.3	1.600 + j 3.936	4.248
3.2	72.01 -j196.8	209.6	1.640 + j 4.481	4.772
3.3	60.59 -j177.4	187.4	1.724 + j 5.048	5.334
3.4	52.92 -j159.8	168.3	1.867 + j 5.639	5.940
3.5	47.88 -j143.7	151.4	2.087 + j 6.264	6.602
3.6	44.92 -j128.7	136.3	2.418 + j 6.927	7.337
3.7	43.34 -j114.5	122.4	2.891 + j 7.638	8.167
3.8	42.96 -j100.5	109.3	3.595 + j 8.410	9.146
3.9	43.61 -j 86.53	96.90	4.645 + j 9.216	10.32
4.0	45.33 -j 72.25	85.29	6.231 + j 9.931	11.72

Table IV

King-Middleton Second-Order Impedances

$$\Omega = 2\ln\frac{2h}{a} = 9 \qquad h/a = 45.01$$

$\beta_o h$	Z_o (ohms)	$\lvert Z_o \rvert$	Y_o (mhos x 10^{-3})	$\lvert Y_o \rvert$ x 10^{-3}
0.5	4.988 -j594.6	594.6	0.014 +j1.681	1.681
0.7	10.31 -j379.8	379.9	0.071 +j2.632	2.633
0.9	18.39 -j245.9	246.6	0.302 +j4.044	4.055
1.1	30.31 -j146.7	149.8	1.351 +j6.537	6.675
1.2	38.29 -j103.6	110.4	3.138 +j8.492	9.053
1.3	48.15 -j 63.05	79.33	7.650 +j10.02	12.60
1.4	60.27 -j 24.07	64.90	14.31 +j5.715	15.41
1.5	75.34 +j 14.00	76.63	12.83 -j2.384	13.05
1.6	94.16 +j 51.69	107.4	8.159 -j4.479	9.308
1.7	118.5 +j 89.54	148.5	5.372 -j4.059	6.733
1.8	150.3 +j126.9	196.7	3.885 -j3.280	5.084
1.9	193.1 +j162.9	252.6	3.026 -j2.552	3.958
2.0	250.6 +j194.0	316.9	2.496 -j1.932	3.156
2.1	327.3 +j213.4	390.8	2.143 -j1.397	2.558
2.2	425.4 +j207.1	473.2	1.900 -j0.925	2.113
2.3	534.9 +j154.7	556.8	1.725 -j0.499	1.796
2.4	624.1 +j 40.64	625.4	1.596 -j0.104	1.599
2.5	646.6 -j117.4	657.2	1.497 +j0.272	1.522
2.6	586.5 -j261.8	642.3	1.422 +j0.635	1.557
2.7	480.0 -j349.1	593.6	1.362 +j0.991	1.684
2.8	371.8 -j379.8	531.5	1.316 +j1.344	1.881
2.9	283.1 -j374.5	469.5	1.284 +j1.699	2.130
3.0	217.0 -j352.7	414.1	1.265 +j2.056	2.414
3.1	169.4 -j324.7	366.2	1.263 +j2.421	2.731
3.2	135.4 -j295.8	325.3	1.280 +j2.796	3.075
3.3	111.3 -j268.1	290.3	1.321 +j3.182	3.445
3.4	94.12 -j242.4	260.0	1.392 +j3.585	3.846
3.5	81.77 -j218.6	233.4	1.501 +j4.013	4.285
3.6	73.05 -j196.4	209.5	1.664 +j4.473	4.773
3.7	66.77 -j175.3	187.6	1.897 +j4.981	5.330
3.8	62.46 -j154.9	167.0	2.239 +j5.554	5.988
3.9	59.86 -j134.8	147.5	2.752 +j6.198	6.782
4.0	58.68 -j114.5	128.6	3.546 +j6.918	7.774

Table V

King-Middleton Second-Order Impedances

$\Omega = 2\ln\frac{2h}{a} = 10 \quad h/a = 75.206$

$\beta_o h$	Z_o(ohms)	$\lvert Z_o\rvert$	Y_o(mhos x 10^{-3})	$\lvert Y_o\rvert$ x 10^{-3}
0.5	4.988 -j704.8	704.8	0.0100 +j1.419	1.419
0.7	10.30 -j451.3	451.5	0.0506 +j2.214	2.215
0.9	18.29 -j293.8	294.4	0.2110 +j3.390	3.397
1.1	30.02 -j177.4	179.9	0.9277 +j5.481	5.559
1.2	37.84 -j127.1	132.6	2.153 +j7.229	7.543
1.3	47.41 -j 79.76	92.79	5.507 +j9.264	10.78
1.4	59.15 -j 34.27	68.36	12.66 +j7.333	14.63
1.5	73.65 +j 10.30	74.37	13.32 -j1.862	13.45
1.6	91.73 +j 54.72	106.8	8.040 -j4.797	9.363
1.7	114.8 +j 99.67	151.3	4.967 -j4.310	6.608
1.8	145.2 +j145.5	205.5	3.437 -j3.444	4.865
1.9	185.5 +j191.8	266.8	2.606 -j2.693	3.748
2.0	240.2 +j237.1	337.5	2.109 -j2.082	2.963
2.1	314.8 +j277.6	419.8	1.787 -j1.576	2.382
2.2	416.1 +j303.8	515.2	1.568 -j1.145	1.941
2.3	547.5 +j296.8	622.8	1.412 -j0.7652	1.606
2.4	697.1 +j227.3	733.3	1.297 -j0.4228	1.364
2.5	820.4 +j 71.37	823.6	1.210 -j0.1052	1.214
2.6	849.9 -j145.6	862.4	1.143 +j0.1958	1.160
2.7	765.5 -j339.9	837.5	1.091 +j0.4846	1.194
2.8	622.2 -j452.6	769.4	1.051 +j0.7646	1.300
2.9	479.4 -j489.4	685.1	1.021 +j1.043	1.460
3.0	364.5 -j480.4	603.1	1.002 +j1.321	1.658
3.1	280.0 -j450.6	530.5	0.9950 +j1.601	1.885
3.2	219.6 -j414.0	468.6	0.9999 +j1.885	2.134
3.3	175.6 -j376.2	415.2	1.019 +j2.182	2.409
3.4	143.8 -j339.9	369.1	1.056 +j2.495	2.709
3.5	120.9 -j306.1	329.1	1.116 +j2.826	3.038
3.6	103.9 -j274.5	293.5	1.207 +j3.186	3.407
3.7	91.22 -j244.7	261.2	1.337 +j3.588	3.829
3.8	81.84 -j216.4	231.3	1.530 +j4.044	4.323
3.9	75.09 -j188.4	202.8	1.825 +j4.580	4.930

Table V

(Continued)

$\beta_o h$	Z_o(ohms)		$\|Z_o\|$	Y_o(mhos x 10^{-3})		$\|Y_o\|$ x 10^{-3}
4.0	70.61	-j160.8	175.6	2.289	+j5.213	5.693
4.1	68.42	-j133.3	149.8	3.049	+j5.938	6.676
4.2	68.61	-j105.4	125.8	4.337	+j6.664	7.951
4.3	71.37	-j 76.93	104.9	6.481	+j6.986	9.529
4.4	77.17	-j 47.77	90.76	9.368	+j5.800	11.02
4.5	86.62	-j 17.91	88.45	11.07	+j2.289	11.31
4.6	100.6	+j 12.49	101.4	9.791	-j1.216	9.867
4.7	120.2	+j 43.02	127.6	7.376	-j2.641	7.834
4.8	147.0	+j 72.76	164.0	5.464	-j2.705	6.097
4.9	182.8	+j 99.87	208.3	4.213	-j2.302	4.801
5.0	229.5	+j121.0	259.4	3.410	-j1.798	3.855
5.1	287.9	+j130.5	316.1	2.882	-j1.306	3.164
5.2	356.3	+j120.5	376.0	2.519	-j0.8516	2.660
5.3	425.9	+j 82.34	433.8	2.263	-j0.4376	2.305
5.4	480.6	+j 12.16	480.8	2.079	-j0.0526	2.080
5.5	502.0	-j 79.40	508.2	1.944	+j0.3074	1.968
5.6	483.1	-j170.1	512.2	1.842	+j0.6486	1.952
5.7	433.4	-j240.3	495.5	1.765	+j0.9786	2.018
5.8	370.0	-j282.3	465.4	1.708	+j1.303	2.148
5.9	307.5	-j299.7	429.4	1.668	+j1.626	2.329
6.0	253.0	-j300.1	392.5	1.642	+j1.948	2.548
6.1	208.4	-j290.3	357.4	1.632	+j2.273	2.798
6.2	173.2	-j275.2	325.2	1.639	+j2.602	3.075
6.3	145.8	-j257.4	295.8	1.666	+j2.941	3.380
6.4	124.7	-j238.8	269.4	1.719	+j3.290	3.712
6.5	108.8	-j220.4	245.8	1.802	+j3.648	4.069
6.6	96.66	-j202.2	224.1	1.924	+j4.026	4.462
6.7	87.46	-j184.4	204.1	2.099	+j4.427	4.899
6.8	80.60	-j167.0	185.4	2.345	+j4.857	5.394
6.9	75.60	-j149.6	167.6	2.690	+j5.324	5.965
7.0	72.19	-j132.0	150.5	3.188	+j5.830	6.645

Table VI

King-Middleton Second-Order Impedances

$\Omega = 2\ln\frac{2h}{a} = 11 \quad h/a = 122.4$

$\beta_o h$	Z_o(ohms)	$\lvert Z_o \rvert$	Y_o(mhos x 10^{-3})	$\lvert Y_o \rvert$ x 10^{-3}
0.5	4.977 -j815.1	815.1	0.007 +j1.227	1.227
0.7	10.30 -j522.9	523.0	0.038 +j1.912	1.912
0.9	18.24 -j341.7	342.2	0.156 +j2.918	2.922
1.1	29.81 -j208.3	210.4	0.673 +j4.704	4.752
1.2	37.51 -j150.5	155.1	1.559 +j6.255	6.447
1.3	46.87 -j 96.47	107.2	4.076 +j8.389	9.327
1.4	58.31 -j 44.56	73.40	10.83 +j8.273	13.63
1.5	72.38 +j 6.378	72.66	13.71 -j1.208	13.76
1.6	89.92 +j 57.21	106.6	7.915 -j5.036	9.381
1.7	112.0 +j109.0	156.3	4.586 -j4.463	6.399
1.8	141.1 +j162.3	215.0	3.051 -j3.509	4.650
1.9	179.2 +j217.6	281.9	2.255 -j2.738	3.547
2.0	231.0 +j274.6	358.9	1.793 -j2.132	2.786
2.1	302.2 +j331.2	448.3	1.503 -j1.648	2.231
2.2	400.7 +j382.2	553.7	1.307 -j1.246	1.806
2.3	537.6 +j413.7	678.4	1.168 -j0.899	1.474
2.4	715.9 +j398.1	819.1	1.067 -j0.593	1.221
2.5	917.1 +j292.0	962.4	0.990 -j0.315	1.039
2.6	1070. +j 64.78	1072.	0.931 -j0.056	0.933
2.7	1080. -j230.7	1104.	0.885 +j0.189	0.905
2.8	939.9 -j471.5	1052.	0.850 +j0.426	0.951
2.9	740.9 -j592.2	948.5	0.824 +j0.658	1.054
3.0	560.2 -j618.5	834.5	0.804 +j0.888	1.198
3.1	421.6 -j594.4	728.8	0.794 +j1.119	1.372
3.2	321.5 -j549.8	636.9	0.793 +j1.356	1.571
3.3	250.6 -j500.0	559.3	0.801 +j1.598	1.787
3.4	199.9 -j451.3	493.6	0.821 +j1.853	2.027
3.5	163.0 -j405.1	436.7	0.855 +j2.124	2.289
3.6	135.7 -j361.4	386.0	0.911 +j2.425	2.590
3.7	115.5 -j321.0	341.2	0.992 +j2.758	2.931
3.8	100.2 -j282.7	299.9	1.114 +j3.142	3.333
3.9	88.98 -j245.9	261.5	1.301 +j3.596	3.776
4.0	81.06 -j209.8	224.9	1.602 +j4.147	4.445

Table VII

King-Middleton Second-Order Impedances

$\Omega = 2\ln\frac{2h}{a} = 12.5 \qquad h/a = 259.01$

$\beta_o h$	Z_o(ohms)	$\lvert Z_o\rvert$	Y_o(mhos x 10^{-3})	$\lvert Y_o\rvert$ x 10^{-3}
0.5	5.005 -j980.6	980.6	0.0052 +j1.020	1.020
0.7	10.29 -j630.3	630.4	0.0259 +j1.586	1.586
0.9	18.18 -j413.6	414.0	0.1061 +j2.413	2.416
1.1	29.60 -j254.2	255.9	0.4520 +j3.881	3.907
1.2	37.13 -j185.7	189.3	1.049 +j5.211	5.315
1.3	46.27 -j121.6	130.1	2.733 +j7.183	7.685
1.4	57.39 -j 60.07	83.08	8.314 +j8.703	12.04
1.5	71.02 +j 0.2681	71.02	14.08 -j0.0532	14.08
1.6	87.90 +j 60.62	106.8	7.718 -j5.322	9.375
1.7	109.2 +j122.2	163.8	4.068 -j4.552	6.104
1.8	136.6 +j186.1	230.8	2.563 -j3.492	4.332
1.9	172.7 +j253.4	306.7	1.837 -j2.694	3.261
2.0	221.2 +j325.0	393.1	1.435 -j2.101	2.544
2.1	287.7 +j400.9	493.5	1.182 -j1.646	2.026
2.2	381.2 +j479.5	612.6	1.016 -j1.278	1.632
2.3	514.5 +j554.2	756.2	0.8997 -j0.9691	1.322
2.4	705.0 +j606.5	930.1	0.8150 -j0.7011	1.075
2.5	965.3 +j594.0	1133.	0.7514 -j0.4624	0.8822
2.6	1268. +j443.0	1344.	0.7025 -j0.2449	0.7440
2.7	1499. +j 93.51	1502.	0.6643 -j0.0414	0.6656
2.8	1490. -j358.7	1533.	0.6343 +j0.1527	0.6524
2.9	1248. -j696.0	1429.	0.6111 +j0.3408	0.6997
3.0	943.6 -j835.5	1260.	0.5941 +j0.5260	0.7934
3.1	689.7 -j841.4	1088.	0.5827 +j0.7108	0.9191
3.2	506.0 -j787.8	936.3	0.5772 +j0.8986	1.068
3.3	378.4 -j714.7	808.7	0.5785 +j1.093	1.236
3.4	289.7 -j640.3	702.8	0.5870 +j1.297	1.424
3.5	227.1 -j569.4	613.0	0.6042 +j1.515	1.631

Table VII
(Continued)

$\beta_o h$	Z_o(ohms)		$\lvert Z_o \rvert$	Y_o(mhos x 10^{-3})		$\lvert Y_o \rvert$ x 10^{-3}
3.6	182.0	-j504.4	536.2	0.6331	+j1.754	1.865
3.7	149.0	-j444.2	468.5	0.6787	+j2.023	2.134
3.8	124.6	-j388.3	407.8	0.7493	+j2.335	2.452
3.9	106.7	-j336.0	352.5	0.8592	+j2.704	2.837
4.0	93.91	-j286.1	301.1	1.036	+j3.155	3.321
4.1	85.39	-j237.9	252.8	1.336	+j3.723	3.955
4.2	80.70	-j191.0	207.3	1.877	+j4.443	4.823
4.3	79.63	-j144.7	165.2	2.919	+j5.304	6.054
4.4	82.23	-j 98.56	128.3	4.993	+j5.982	7.792
4.5	88.77	-j 52.14	103.0	8.375	+j4.919	9.713
4.6	99.77	-j 5.109	99.90	9.997	+j0.5119	10.01
4.7	116.0	+j 42.88	123.7	7.584	-j2.802	8.085
4.8	138.8	+j 91.94	166.5	5.006	-j3.316	6.005
4.9	169.9	+j141.9	221.4	3.466	-j2.895	4.516
5.0	211.8	+j192.2	286.1	2.589	-j2.348	3.495
5.1	268.1	+j241.1	360.6	2.062	-j1.854	2.773
5.2	342.9	+j284.5	445.6	1.727	-j1.433	2.244
5.3	440.5	+j315.0	541.5	1.502	-j1.074	1.846
5.4	562.7	+j319.5	647.1	1.344	-j0.7629	1.545
5.5	703.2	+j278.7	756.4	1.229	-j0.4871	1.322
5.6	838.4	+j174.0	856.3	1.143	-j0.2373	1.168
5.7	927.2	+j 5.512	927.2	1.078	-j0.0064	1.078
5.8	933.5	-j191.0	952.8	1.029	+j0.2107	1.050
5.9	856.6	-j361.6	929.8	0.9908	+j0.4183	1.075
6.0	734.5	-j472.9	873.6	0.9625	+j0.6197	1.145
6.1	605.1	-j525.0	801.1	0.9429	+j0.8180	1.248
6.2	490.3	-j534.5	725.3	0.9317	+j1.016	1.378
6.3	396.7	-j519.1	653.3	0.9293	+j1.216	1.530
6.4	323.1	-j490.5	587.3	0.9367	+j1.422	1.703
6.5	266.3	-j455.9	527.9	0.9554	+j1.636	1.894
6.6	222.1	-j419.3	474.5	0.9869	+j1.862	2.108
6.7	187.7	-j382.2	425.8	1.035	+j2.108	2.348
6.8	160.7	-j345.5	381.0	1.107	+j2.379	2.624
6.9	139.6	-j309.5	339.5	1.211	+j2.685	2.945
7.0	123.1	-j273.7	300.1	1.367	+j3.039	3.333

Table VIII

King-Middleton Second-Order Impedances

$\Omega = 2\ln \frac{2h}{a} = 15 \quad h/a = 904.02$

$\beta_o h$	Z_o(ohms)	$\lvert Z_o \rvert$	Y_o(hmos x 10^{-3})	$\lvert Y_o \rvert$ x 10^{-3}
0.5	5.000-j1256.	1256.	0.0032 +j0.7960	0.7960
0.7	10.28 -j 809.3	809.3	0.0157 +j1.236	1.236
0.9	18.13 -j 533.2	533.5	0.0637 +j1.873	1.874
1.1	29.36 -j 330.9	332.2	0.2660 +j2.998	3.010
1.2	36.73 -j 244.4	247.1	0.6014 +j4.001	4.046
1.3	45.62 -j 163.5	169.7	1.584 +j5.676	5.892
1.4	56.38 -j 86.00	102.8	5.331 +j8.132	9.724
1.5	69.46 -j 10.23	70.21	14.09 +j2.076	14.24
1.6	85.53 +j 65.50	107.7	7.369 -j5.644	9.282
1.7	105.7 +j 142.8	177.7	3.348 -j4.525	5.629
1.8	131.5 +j 223.4	259.2	1.956 -j3.324	3.857
1.9	165.0 +j 309.1	350.3	1.344 -j2.518	2.854
2.0	209.4 +j 401.7	453.0	1.021 -j1.957	2.208
2.1	269.8 +j 503.3	571.0	0.8273 -j1.543	1.751
2.2	354.8 +j 616.3	711.4	0.7016 -j1.219	1.406
2.3	476.3 +j 740.3	880.3	0.6146 -j0.9553	1.136
2.4	656.3 +j 871.2	1091.	0.5518 -j0.7337	0.9180
2.5	929.4 +j 988.5	1353.	0.5047 -j0.5397	0.7389
2.6	1330. +j1045.	1691.	0.4684 -j0.3639	0.5932
2.7	1878. +j 860.4	2066.	0.4401 -j0.2017	0.4841
2.8	2361. +j 278.9	2378.	0.4177 -j0.0493	0.4206
2.9	2362. -j 570.8	2430.	0.4000 +j0.0967	0.4115
3.0	1870. -j1159.	2200.	0.3864 +j0.2393	0.4545
3.1	1312. -j1328.	1705.	0.3765 +j0.3809	0.5356
3.2	898.8 -j1273.	1558.	0.3702 +j0.5242	0.6417
3.3	626.4 -j1146.	1306.	0.3674 +j0.6720	0.7659
3.4	449.8 -j1008.	1104.	0.3689 +j0.8271	0.9050
3.5	332.8 -j 881.1	941.9	0.3752 +j0.9932	1.062
3.6	253.2 -j 767.3	808.0	0.3878 +j1.175	1.238
3.7	197.6 -j 666.1	694.8	0.4093 +j1.380	1.439
3.8	158.1 -j 574.7	596.0	0.4450 +j1.616	1.676
3.9	129.9 -j 492.9	509.8	0.4998 +j1.897	1.962

Table VIII
(Continued)

$\beta_o h$	Z_o(ohms)	$\|Z_o\|$	Y_o(hmos x 10^{-3})	$\|Y_o\| \times 10^{-3}$
4.0	110.0 -j 417.0	431.2	0.5913 +j2.242	2.319
4.1	97.25 -j 345.9	359.3	0.7380 +j2.646	2.747
4.2	89.70 -j 278.2	292.3	1.036 +j3.267	3.427
4.3	85.53 -j 212.5	229.1	1.630 +j4.049	4.365
4.4	85.48 -j 148.5	171.3	2.881 +j5.056	5.819
4.5	89.91 -j 85.51	124.1	5.837 +j5.509	8.026
4.6	99.14 -j 22.67	101.7	9.537 +j2.218	9.792
4.7	113.5 +j 41.25	120.7	7.784 -j2.830	8.282
4.8	133.9 +j 107.2	171.5	4.551 -j3.644	5.830
4.9	161.9 +j 175.7	238.9	2.840 -j3.079	4.189
5.0	199.8 +j 247.1	316.4	1.979 -j2.447	3.147
5.1	250.9 +j 321.7	408.0	1.507 -j1.933	2.451
5.2	320.2 +j 399.2	511.7	1.223 -j1.524	1.954
5.3	414.5 +j 476.5	631.6	1.037 -j1.195	1.583
5.4	543.5 +j 547.5	771.5	0.9132 -j0.9199	1.296
5.5	717.7 +j 597.1	933.6	0.8234 -j0.6850	1.071
5.6	944.4 +j 596.8	1117.	0.7566 -j0.4782	0.8951
5.7	1210. +j 500.0	1309.	0.7060 -j0.2919	0.7640
5.8	1452. +j 262.2	1475.	0.6672 -j0.1205	0.6780
5.9	1564. -j 99.34	1567.	0.6368 +j0.0404	0.6381
6.0	1481. -j 469.4	1554.	0.6135 +j0.1944	0.6436
6.1	1258. -j 726.5	1453.	0.5961 +j0.3443	0.6884
6.2	1001. -j 843.2	1308.	0.5839 +j0.4925	0.7639
6.3	775.0 -j 862.1	1159.	0.5767 +j0.6415	0.8626
6.4	598.4 -j 826.7	1020.	0.5746 +j0.7937	0.9798
6.5	465.6 -j 766.8	897.1	0.5784 +j0.9524	1.114
6.6	367.4 -j 698.8	789.5	0.5893 +j1.121	1.266
6.7	293.8 -j 629.8	695.0	0.6083 +j1.304	1.439
6.8	238.5 -j 562.5	611.0	0.6389 +j1.507	1.637
6.9	196.6 -j 497.9	535.3	0.6860 +j1.738	1.868
7.0	164.7 -j 436.0	466.0	0.7584 +j2.007	2.146

Table IX

King-Middleton Second-Order Impedances

$\Omega = 2\ln\frac{2h}{a} = 20 \quad h/a = 11013$

$\beta_o h$	Z_o(ohms)	$\lvert Z_o \rvert$	Y_o(mhos x 10^{-3})	$\lvert Y_o \rvert$ x 10^{-3}
0.5	5.030-j1809.	1809.	0.0015 +j0.5527	0.5527
0.9	18.93 -j 885.4	885.6	0.0241 +j1.129	1.129
1.3	44.92 -j 247.3	251.3	0.7110 +j3.914	3.978
1.4	55.22 -j 138.0	148.6	2.499 +j6.246	6.728
1.5	67.65 -j 31.59	74.66	12.14 +j5.667	13.40
1.6	82.98 +j 74.58	111.6	6.666 -j5.991	8.963
1.7	101.9 +j 182.9	209.4	2.325 -j4.172	4.776
1.8	125.3 +j 295.1	320.6	1.219 -j2.871	2.022
1.9	155.5 +j 414.3	442.5	0.7941 -j2.116	2.260
2.0	195.3 +j 544.3	578.3	0.5840 -j1.628	1.729
2.1	247.7 +j 688.9	732.1	0.4537 -j1.285	1.363
2.2	320.0 +j 853.0	911.0	0.3855 -j1.028	1.098
2.3	432.1 +j1043.	1129.	0.3338 -j0.8231	0.8857
2.4	576.0 +j1269.	1394.	0.2966 -j0.6543	0.7174
2.5	815.1 +j1538.	1741.	0.2689 -j0.5075	0.5744
2.6	1210. +j1850.	2211.	0.2475 -j0.3786	0.4523
2.7	1895. +j2150.	2866.	0.2307 -j0.2617	0.3488
2.8	3075. +j2165.	3761.	0.2174 -j0.1531	0.2659
2.9	4567. +j1104.	4699.	0.2068 -j0.0500	0.2128
3.0	4790. -j1191.	4888.	0.1984 +j0.0498	0.2046
3.1	3262. -j2521.	4123.	0.1919 +j0.1483	0.2425
3.2	1944. -j2570.	3223.	0.1872 +j0.2475	0.3103
3.3	1182. -j2240.	2533.	0.1842 +j0.3493	0.3949
3.4	759.3 -j1890.	2037.	0.1831 +j0.4557	0.4911
3.5	514.3 -j1591.	1671.	0.1892 +j0.5693	0.5999
3.6	364.1 -j1343.	1392.	0.1880 +j0.6935	0.7185
3.7	267.5 -j1138.	1170.	0.1956 +j0.8325	0.8552
3.8	203.1 -j 964.9	986.2	0.2089 +j0.9924	1.014
3.9	159.4 -j 814.9	830.4	0.2311 +j1.182	1.204
4.0	130.4 -j 682.2	694.5	0.2707 +j1.414	1.440
4.1	109.3 -j 562.4	573.1	0.3329 +j1.713	1.745
4.2	96.62 -j 452.2	462.4	0.4519 +j2.115	2.163
4.3	89.81 -j 348.6	360.1	0.6929 +j2.690	2.778
4.4	88.04 -j 249.7	264.8	1.256 +j3.562	3.778

Table IX
(Continued)

$\beta_o h$	Z_o(ohms)	$\|Z_o\|$	Y_o(mhos x 10^{-3})	$\|Y_o\| \times 10^{-3}$
4.5	90.90 -j 153.5	178.4	2.846 +j4.817	5.596
4.6	98.33 -j 58.47	114.4	7.512 +j4.467	8.740
4.7	110.6 +j 37.10	116.6	8.127 -j2.726	8.572
4.8	128.5 +j 134.8	186.2	3.706 -j3.887	5.371
4.9	153.1 +j 236.2	281.6	1.932 -j2.981	3.552
5.0	186.2 +j 343.4	390.7	1.220 -j2.250	2.560
5.1	230.8 +j 458.5	513.4	0.8761 -j1.740	1.948
5.2	291.1 +j 583.8	652.4	0.6841 -j1.372	1.933
5.3	373.8 +j 721.8	813.1	0.5657 -j1.092	1.230
5.4	489.7 +j 875.0	1003.	0.4871 -j0.8703	0.9973
5.5	656.1 +j1043.	1233.	0.4320 -j0.6868	0.8114
5.6	901.4 +j1220.	1517.	0.3917 -j0.5302	0.6592
5.7	1270. +j1379.	1875.	0.3613 -j0.3925	0.5334
5.8	1815. +j1442.	2318.	0.3378 -j0.2683	0.4314
5.9	2542. +j1222.	2821.	0.3196 -j0.1536	0.3545
6.0	3215. +j 447.8	3247.	0.3053 -j0.0455	0.3086
6.1	3273. -j 651.4	3337.	0.2940 +j0.0585	0.2998
6.2	2663. -j1497.	3055.	0.2854 +j0.1604	0.3274
6.3	1904. -j1786.	2612.	0.2793 +j0.2620	0.3829
6.4	1314. -j1744.	2199.	0.2755 +j0.3658	0.4581
6.5	916.4 -j1582.	1828.	0.2743 +j0.4734	0.5471
6.6	656.2 -j1395.	1542.	0.2760 +j0.5869	0.6485
6.7	482.1 -j1217.	1310.	0.2811 +j0.7100	0.7636
6.8	363.2 -j1056.	1117.	0.2910 +j0.8465	0.8951
6.9	280.2 -j 912.7	954.9	0.3074 +j1.001	1.048
7.0	221.3 -j 786.3	816.4	0.3316 +j1.178	1.224

Table X

Tables of Impedance* of Cylindrical Antenna of Constant Radius-to-Wavelength Ratio

h = half-length of antenna ; a = radius of antenna

$a/\lambda_o = .00119$

βh	Z_o (ohms)		Z_o
1.1	29.73	-j219.5	221.5
1.2	37.38	-j165.7	169.8
1.3	46.58	-j108.2	117.8
1.4	57.77	-j 53.1	78.47
1.5	71.42	+j 2.50	71.46
1.6	88.35	+j 54.7	103.9
1.7	109.6	+j120.2	162.6
1.8	136.9	+j183.8	229.2
1.9	172.8	+j252.4	305.9
2.0	218.0	+j326.9	392.9
2.1	281.	+j406.	494.
2.2	380.	+j495.	624.
2.3	509.	+j581.	772.
2.4	698.	+j658.	959.
2.5	961.	+j689.	1182.
2.6	1298.	+j606.	1432.
2.7	1639.	+j295.	1665.
2.8	1760.	-j234.	1775.
2.9	1580.	-j687.	1723.
3.0	1234.	-j959.	1563.
3.1	905.	-j1018.	1362.
3.2	654.	-j976.	1175.
3.3	480.	-j890.	1011.
3.4	350.	-j793.	867.
3.5	277.	-j714.	766.
3.6	218.	-j634.	670.
3.7	176.	-j558.	585.
3.8	143.	-j488.	508.
3.9	122.	-j422.	439.
4.0	104.	-j363.	378.

For λ_o=100 cm. d = 3/32"

$a/\lambda_o = .00158$

βh	Z_o (ohms)		Z_o
1.1	29.86	-j201.7	203.9
1.2	37.52	-j149.8	154.4
1.3	46.82	-j 98.8	109.3
1.4	58.13	-j 47.2	74.88
1.5	71.94	+j 4.8	72.10
1.6	89.10	+j 58.5	106.6
1.7	110.6	+j115.0	159.6
1.8	138.3	+j174.9	223.0
1.9	174.8	+j239.0	296.1
2.0	223.4	+j309.0	380.5
2.1	289.	+j382.	479.
2.2	389.	+j461.	603.
2.3	518.	+j533.	743.
2.4	709.	+j586.	920.
2.5	960	+j584.	1124.
2.6	1263.	+j445.	1339.
2.7	1518.	+j123.	1523.
2.8	1549.	-j336.	1585.
2.9	1338.	-j698.	1509.
3.0	1045.	-j881.	1367.
3.1	780.	-j910.	1198.
3.2	570.	-j868.	1038.
3.3	425.	-j795.	901.
3.4	319.	-j713.	781.
3.5	253.	-j643.	691.
3.6	202.	-j573.	608.
3.7	164.	-j508.	534.
3.8	135.	-j445.	465.
3.9	116.	-j385.	402.
4.0	100.	-j333.	348.

For λ_o=100 cm. d = 1/8"

*These values are only accurate to within three figures in the third digit.

Table X

(cont.- 1)

a/λ_o = .00238			a/λ_o = .00318		
βh	Z_o(ohms)	Z_o	βh	Z_o(ohms)	Z_o
1.1	30.07 -j176.9	179.4	1.1	30.20 -j159.2	162.0
1.2	37.79 -j130.8	136.2	1.2	38.01 -j117.4	123.4
1.3	47.26 -j 85.1	97.34	1.3	47.60 -j 75.5	89.25
1.4	58.72 -j 38.8	70.38	1.4	59.23 -j 33.3	67.95
1.5	72.80 +j 7.9	73.22	1.5	73.59 +j 10.2	74.29
1.6	90.35 +j 56.5	106.5	1.6	91.42 +j 55.1	106.7
1.7	112.4 +j107.8	155.7	1.7	113.9 +j102.5	153.2
1.8	141.2 +j161.7	214.7	1.8	143.5 +j151.4	208.6
1.9	178.9 +j219.9	283.5	1.9	182.3 +j204.9	274.3
2.0	229.8 +j281.0	363.0	2.0	234.3 +j259.0	350.1
2.1	300. +j344.	456.	2.1	305. +j315.	438.
2.2	398. +j408.	570.	2.2	403. +j367.	545.
2.3	529. +j457.	699.	2.3	537. +j400.	648.
2.4	718. +j476.	862.	2.4	717. +j392.	817.
2.5	944. +j418.	1032.	2.5	921. +j300.	969.
2.6	1179. +j231.	1201.	2.6	1091. +j 90.0	1095.
2.7	1310. -j 86.0	1313.	2.7	1140. -j196.	1157.
2.8	1245. -j426.	1316.	2.8	1020. -j464.	1121.
2.9	1037. -j664.	1231.	2.9	845. -j621.	1049.
3.0	805. -j764.	1110.	3.0	656. -j678.	943.
3.1	602. -j764.	973.	3.1	500. -j670.	836.
3.2	454. -j726.	856.	3.2	389. -j634.	744.
3.3	348. -j668.	753.	3.3	298. -j584.	656.
3.4	271. -j607.	665.	3.4	240. -j533.	584.
3.5	218. -j549.	591.	3.5	193. -j483.	520.
3.6	178. -j491.	518.	3.6	160. -j435.	463.
3.7	147. -j438.	462.	3.7	134. -j390.	412.
3.8	124. -j384.	404.	3.8	114. -j344.	362.
3.9	108. -j335.	352.	3.9	102. -j302.	319.
4.0	93.8 -j289.	304.	4.0	89.7 -j260.	275.
For λ_o = 100 cm, d = 3/16"			For λ_o = 100 cm, d = 1/4"		

Table X
(cont.- 2)

a/λ_o = .00397				a/λ_o = .00476			
βh	Z_o(ohms)		Z_o	βh	Z_o(ohms)		Zo
1.1	30.35	-j145.6	148.7	1.1	30.48	-j134.5	138.
1.2	38.20	-j106.8	113.4	1.2	38.41	-j 98.2	105.
1.3	47.92	-j 67.7	82.9	1.3	48.23	-j 61.8	78.4
1.4	59.72	-j 28.6	66.2	1.4	60.15	-j 24.9	65.1
1.5	74.26	+j 11.9	75.2	1.5	74.94	+j 13.3	76.1
1.6	92.37	+j 53.7	106.9	1.6	93.28	+j 52.7	107.
1.7	115.3	+j 97.9	151.3	1.7	116.7	+j 94.2	150.
1.8	145.4	+j144.3	204.8	1.8	147.0	+j138.3	202.
1.9	185.2	+j193.0	267.5	1.9	187.7	+j183.2	262.
2.0	238.8	+j243.2	340.9	2.0	242.4	+j228.8	333.
2.1	310.	+j291.	425.	2.1	314.	+j271.	416.
2.2	409.	+j332.	527.	2.2	415.	+j303.	514.
2.3	542.	+j351.	646.	2.3	545.	+j307	626.
2.4	710.	+j322.	780.	2.4	700.	+j255.	745.
2.5	888.	+j203.	911.	2.5	850.	+j126.	859.
2.6	1005.	+j 10.1	1005.	2.6	929.	-j 84.0	933.
2.7	1001.	-j260.	1034.	2.7	884.	-j303.	934.
2.8	885.	-j473.	1003.	2.8	763.	-j469.	896.
2.9	713.	-j582.	920.	2.9	613.	-j544.	820.
3.0	555.	-j615.	828.	3.0	478.	-j563.	738.
3.1	426.	-j598.	734.	3.1	370.	-j546.	660.
3.2	333.	-j566.	657.	3.2	292.	-j514.	591.
3.3	260.	-j523.	584.	3.3	233.	-j476.	530.
3.4	214.	-j478.	524.	3.4	191.	-j434.	474.
3.5	174.	-j433.	466.	3.5	159.	-j396.	427.
3.6	146.	-j392.	418.	3.6	133.	-j359.	383.
3.7	124.	-j353.	374.	3.7	115.	-j323.	343.
3.8	108.	-j313.	332.	3.8	102.	-j288.	305.
3.9	96.2	-j275.	291.	3.9	90.7	-j253.	269.
4.0	85.6	-j236.	251.	4.0	82.7	-j218.	233.

For λ_o = 100 cm, d = 5/16" For λ_o = 100 cm, d = 3/8"

Table X
(cont. - 3)

$a/\lambda_o = .00635$			$a/\lambda_o = .00952$		
βh	Z_o(ohms)	$\|Z_o\|$	βh	Z_o(ohms)	$\|Z_o\|$
1.1	30.70 -j116.6	121.	1.1	31.19 -j 91.4	96.6
1.2	38.87 -j 84.7	93.2	1.2	39.48 -j 65.7	76.6
1.3	48.77 -j 52.2	71.4	1.3	49.75 -j 38.7	63.0
1.4	60.99 -j 19.1	63.9	1.4	62.37 -j 11.2	63.4
1.5	76.06 +j 15.3	77.6	1.5	78.02 +j 17.6	80.0
1.6	94.83 +j 50.8	108.	1.6	97.58 +j 47.7	109.
1.7	118.9 +j 88.1	148.	1.7	122.6 +j 78.8	146.
1.8	150.3 +j126.2	196.	1.8	155.8 +j110.0	191.
1.9	192.3 +j165.7	254.	1.9	199.5 +j139.3	243.
2.0	250.0 +j203.9	323.	2.0	260.0 +j163.0	307.
2.1	320. +j234.	396.	2.1	330. +j174.	373.
2.2	420. +j251.	489.	2.2	421. +j160.	450.
2.3	542. +j228.	588.	2.3	522. +j104.	532.
2.4	674. +j150.	690.	2.4	596. +j 2.50	596.
2.5	764. +j 5.00	764.	2.5	612. -j140.	628.
2.6	786. -j184.	807.	2.6	563. -j264.	622.
2.7	703. -j349.	785.	2.7	480. -j349.	594.
2.8	598. -j446.	746.	2.8	392. -j388.	552.
2.9	472. -j484.	676.	2.9	310. -j393.	500.
3.0	370. -j483.	608.	3.0	250. -j377.	452.
3.1	290. -j466.	549.	3.1	192. -j363.	411.
3.2	233. -j437.	495.	3.2	162. -j339.	376.
3.3	190. -j404.	446.	3.3	136. -j310.	338.
3.4	156. -j368.	400.	3.4	115. -j285.	307.
3.5	134. -j338.	364.	3.5	102. -j264.	283.
3.6	115. -j308.	329.	3.6	90.0 -j240.	256.
3.7	102. -j279.	297.	3.7	82.5 -j218.	233.
3.8	91.3 -j248.	264.	3.8	75.5 -j196.	210.
3.9	82.8 -j219.	234.	3.9	71.0 -j174.	188.
4.0	77.2 -j189.	204.	4.0	68.7 -j151.	166.
For λ_o = 100 cm, d = 1/2"			For λ_o = 100 cm, d = 3/4"		

Table XI

Values of Ω

$$\Omega = 2 \log \frac{\beta h}{\pi/2} + 2 \log \frac{\lambda}{d}$$

λ = 100 cm for the values of d listed

βh	d = 3/32"	d = 1/8"	d = 3/16"	d = 1/4"
1.1	11.36774	10.79236	9.98138	9.40606
1.2	11.54176	10.96638	10.15540	9.58008
1.3	11.70186	11.12648	10.31550	9.74018
1.4	11.85008	11.27470	10.46372	9.88840
1.5	11.98806	11.41268	10.60170	10.02638
1.6	12.11714	11.54176	10.73078	10.15546
1.7	12.23828	11.66290	10.85192	10.27660
1.8	12.35266	11.77728	10.96630	10.39098
1.9	12.46086	11.88548	11.07450	10.49918
2.0	12.56334	11.98796	11.17698	10.60166
2.1	12.66098	12.08560	11.27462	10.69930
2.2	12.75408	12.17870	11.36772	10.79240
2.3	12.84290	12.26752	11.45654	10.88122
2.4	12.92806	12.35268	11.54170	10.96638
2.5	13.00964	12.43426	11.62328	11.04796
2.6	13.08812	12.51274	11.70176	11.12644
2.7	13.16364	12.58826	11.77728	11.20196
2.8	13.23632	12.66094	11.84996	11.27464
2.9	13.30654	12.73116	11.92018	11.34486
3.0	13.37438	12.79900	11.98802	11.41270
3.1	13.43990	12.86452	12.05354	11.47822
3.2	13.50344	12.92806	12.11708	11.54176
3.3	13.56492	12.98954	12.17856	11.60324
3.4	13.62466	13.04928	12.23830	11.66298
3.5	13.68266	13.10728	12.29630	11.72098
3.6	13.73896	13.16358	12.35260	11.77728
3.7	13.79378	13.21840	12.40742	11.83210
3.8	13.84716	13.27178	12.46080	11.88548
3.9	13.89906	13.32368	12.51270	11.93738
4.0	13.94972	13.37434	12.56336	11.98804

Table XI

Values of Ω (continued)

$$\Omega = 2 \log \frac{\beta h}{\pi/2} + 2 \log \frac{\lambda}{d}$$

λ = 100 cm for the values of d listed

βh	d = 5/16"	d = 3/8"	d = 1/2"	d = 3/4"
1.1	8.95970	8.59518	8.01976	7.20880
1.2	9.13372	8.76920	8.19378	7.38282
1.3	9.29382	8.92930	8.35388	7.54292
1.4	9.44204	9.07752	8.50210	7.69114
1.5	9.58002	9.21550	8.64008	7.82912
1.6	9.70910	9.34458	8.76916	7.95820
1.7	9.83024	9.46572	8.89030	8.07934
1.8	9.94462	9.58010	9.00468	8.19372
1.9	10.05282	9.68830	9.11288	8.30192
2.0	10.15530	9.79078	9.21536	8.40440
2.1	10.25294	9.88842	9.31300	8.50204
2.2	10.34604	9.98152	9.40610	8.59514
2.3	10.43486	10.07034	9.49492	8.68396
2.4	10.52002	10.15550	9.58008	8.76912
2.5	10.60160	10.23708	9.66166	8.85070
2.6	10.68008	10.31556	9.74014	8.92918
2.7	10.75560	10.39108	9.81566	9.00470
2.8	10.82828	10.46376	9.88834	9.07738
2.9	10.89850	10.53398	9.95856	9.14760
3.0	10.96634	10.60182	10.02640	9.21544
3.1	11.03186	10.66734	10.09192	9.28096
3.2	11.09540	10.73088	10.15546	9.34450
3.3	11.15688	10.79236	10.21694	9.40598
3.4	11.21662	10.85210	10.27668	9.46572
3.5	11.27462	10.91010	10.33468	9.52372
3.6	11.33092	10.96640	10.39098	9.58002
3.7	11.38574	11.02122	10.44580	9.63484
3.8	11.43912	11.07460	10.49918	9.68822
3.9	11.49102	11.12650	10.55108	9.74012
4.0	11.54168	11.17716	10.60174	9.79078

Table XII

Additional Values of Z_o Near Antiresonance

$\beta_o h$	$\Omega = 7$	$\Omega = 8$	$\Omega = 9$	$\Omega = 10$
2.00	264.0 +j 69.90			
2.05	291.4 +j 52.98			
2.10	315.9 +j 27.76	334.1 +j131.5		
2.15	334.9 -j 5.498	373.5 +j114.3		
2.20	344.6 -j 44.82	411.6 +j 86.47	425.4 +j207.1	
2.25	343.4 -j 86.82	445.1 +j 47.07	480.3 +j188.1	
2.30	330.5 -j127.0	469.4 -j 2.858	534.9 +j154.7	547.5 +j296.8
2.35	308.0 -j161.3	479.9 -j 59.87	584.9 +j105.3	621.9 +j271.9
2.40	279.0 -j187.6	474.3 -j118.5	624.1 +j 40.64	697.1 +j227.3
2.45	247.1 -j205.0	453.8 -j173.1	646.1 -j 36.01	766.7 +j160.4
2.50	215.2 -j214.4	421.2 -j218.4	646.6 -j117.4	820.4 +j 71.37
2.55		381.5 -j252.2	625.3 -j195.3	850.3 -j 33.55
2.60		338.9 -j274.3	586.5 -j261.8	849.9 -j145.6
2.65			535.6 -j313.7	819.6 -j250.9
2.70			480.0 -j349.1	765.5 -j339.9
2.75				696.6 -j407.3
2.80				622.2 -j452.6

$\beta_o h$	$\Omega = 11$	$\Omega = 12.5$	$\Omega = 15$	$\Omega = 20$
2.40	715.9 +j398.1			
2.45	816.9 +j359.1			
2.50	917.1 +j292.0	965.3 +j594.0		
2.55	1007. +j192.4	1116. +j540.9		
2.60	1070. +j 64.78	1268. +j443.0	1330.+j1045	
2.65	1098. -j 82.37	1405. +j291.9	1593.+j 987.8	
2.70	1080. -j230.7	1499. +j 93.51	1878.+j 860.4	1895.+j2150.
2.75	1025. -j364.6	1531. -j133.6	2153.+j 626.0	2413.+j2228.
2.80	939.9 -j471.5	1490. -j358.7	2361.+j 278.9	3075.+j2165.
2.85	842.1 -j546.0	1388. -j552.1	2438.-j 166.7	3847.+j1834.
2.90	740.9 -j592.2	1248. -j696.0	2362.-j 570.9	4567.+j1104.
2.95		1094. -j787.9	2149.-j 924.4	4943.-j 5.4
3.00		943.6 -j835.5	1870.-j1159.	4790.-j1191.
3.05			1579.-j1285.	4076.-j2071.
3.10			1313.-j1328.	3262.-j2521.
3.15				2527.-j2639.
3.20				1944.-j2570.

Table XIII

Critical Quantities in King-Middleton Second-Order Impedance $Z_o = R_o + jX_o$

Ω	n	$\beta_o h$ anti-res.	R_o anti-res.	R_o max	X_o max	X_o min	$\frac{\lvert X_o \rvert_{min}}{\lvert X_o \rvert_{max}}$	n	$\beta_o h$ res.	R_o res.	R_o $\beta_o h = \frac{n\pi}{2}$	X_o $\beta_o h = \frac{n\pi}{2}$
7	2	2.13	321.6	344.6	83.6	215.2	2.574	1	1.419	67.25	95.68	35.85
8	2	2.30	469.4	474.3	140.7	290.3	2.063	1	1.445	68.92	91.62	38.98
9	2	2.43	630.8	646.6	213.4	379.8	1.780	1	1.463	69.76	88.66	40.68
10	2 4	2.54 5.42	844 484	855 500	305 130	492 302	1.613 2.325	1 3	1.477 4.560	70.3 94.0	86.5 122.5	41.7 46.0
11	2	2.62	1072	1095	413.7	618.5	1.495	1	1.487	70.55	84.80	42.36
12.5	2 4	2.72 5.70	1520 928	1531 942	615 324	850 535	1.381 1.650	1 3	1.504 4.615	71.0 101.2	83.0 116.6	43.0 46.5
15	2 4	2.83 5.87	2430 1555	2438 1565	1050 610	1335 865	1.271 1.418	1 3	1.514 4.636	71.7 103.2	80.8 114.0	43.4 46.5
20	2 4	2.95 6.04	4940 3330	4950 3340	2230 1446	2640 1800	1.181 1.245	1 3	1.530 4.662	72.2 104.6	78.5 111.5	43.6 46.5

TABLES XIII (Continued)

Critical Quantities in King - Middleton Second-Order Admittances $Y_o = G_o + jB_o^1$

Ω	n	$\beta_o h$ res	G_o res $\times 10^{-3}$	G_o max $\times 10^{-3}$	B_o max $\times 10^{-3}$	B_o min $\times 10^{-3}$	$\frac{B_o\ max}{B_o\ min}$	G_o $\beta_o h = \frac{n\pi}{2}$ $\times 10^{-3}$	B_o $\beta_o h = \frac{n\pi}{2}$ $\times 10^{-3}$	n	$\beta_o h$ antires	G_o antires $\times 10^{-3}$
7	1	1.419	14.87	16.10	11.80	3.50	3.37	9.16	3.43	2	2.13	3.11
8	1	1.445	14.51	15.42	10.60	4.08	2.60	9.24	3.93	2	2.30	2.13
9	1	1.463	14.33	14.90	10.00	4.50	2.22	9.32	4.28	2	2.43	1.58
10	1	1.477	14.22	14.66	9.53	4.81	1.98	9.38	4.52	2	2.54	1.11
	3	4.560	10.64	11.09	6.98	2.79	2.50	7.15	2.69	4	5.42	2.07
11	1	1.487	14.17	14.50	9.15	5.00	1.83	9.44	4.71	2	2.62	0.933
12.5	1	1.504	14.08	14.30	8.80	5.33	1.65	9.50	4.92	2	2.72	0.658
	3	4.615	9.88	10.06	6.00	3.35	1.79	7.40	2.95	4	5.70	1.08
15	1	1.514	13.95	14.10	8.37	5.63	1.49	9.60	5.16	2	2.83	0.412
	3	4.636	9.69	9.70	5.58	3.70	1.51	7.52	3.07	4	5.87	0.643
20	1	1.530	13.85	13.96	7.90	6.00	1.32	9.74	5.41	2	2.95	0.202
	3	4.662	9.56	9.58	5.29	4.05	1.31	7.64	3.19	4	6.04	0.300

TABLE XIV

Comparison Of Second-and Third-Order Conductances and Resistances At $\beta_o h = \pi/2$

$\Omega = 2\ln\frac{2h}{a}$	$[G_o]_3$ MHOS	$[G_o]_2$ MHOS	% difference of second-order conductance	$[R_o]_3$ OHMS	$[R_o]_2$ OHMS	% difference of second-order resistance
7	8.35×10^{-3}	9.16×10^{-3}	8.84	107.9	95.7	12.8
8	8.73	9.24	5.52	99.3	91.6	8.4
9	8.97	9.32	3.50	93.8	88.7	5.7
10	9.15	9.38	2.45	89.9	86.5	3.9
12.5	9.41	9.50	.947	84.3	83.0	1.6
15	9.56	9.61	.520	81.5	80.8	.87
20	9.76	9.74	—	78.1	78.5	—

Table XV

$\Omega = 10$

Resistance and Reactance of Short Antenna

$\beta_o h$	X_o (ohms)			R_o (ohms)			
	Formula for Short Antenna	K.M. Second-Order Using Ψ_{K1}	Inter-polated	Formula for Short Antenna	K.M. Second-Order Using Ψ_{K1}	Inter-polated	$20\beta_o^2 h^2(1 + \frac{2}{15}\beta^2 h^2)$
0	$-\infty$	$-\infty$	$-\infty$	0	0	0	0
0.1	-3945		-3945	0.183		0.183	0.200
0.2	-1950		-1950	0.732		0.732	0.804
0.3	-1274		-1274	1.66		1.66	1.82
0.4	- 929		- 929	2.97		2.97	3.27
0.5	- 716	-704.8	- 716	4.67	4.99	4.67	5.17
0.6	- 569		- 569	6.80		6.80	7.54
0.7	- 460	-451.3	- 460	9.35	10.30	9.4	10.44
0.8	- 373.7		- 370	12.38		12.5	13.89
0.9	- 303.5	-293.8	- 294	15.82	18.3	16.2	17.95
1.0	- 238.2		- 234	19.90		21.0	22.67
1.1		-177.4	- 177.4		30.0	27.3	28.10
1.2		-127.1	- 127.1		37.8	35.4	34.33
1.3		- 79.8	- 79.8		47.4	45.6	41.41
1.4		- 34.3	- 34.3		59.1	58.2	
1.5		+ 10.3	+ 10.3		73.6	73.6	
1.6		+ 54.7	+ 54.7		91.7	91.7	
1.7		+ 99.7	+ 99.7		114.8	114.8	

Table XVI

TABULATION OF MEASURED IMPEDANCES

$\lambda = 60$ cm.

$\frac{a}{\lambda} = 2.98 \times 10^{-3}$

RESISTANCES					
$\beta_o h$	$\frac{b}{a} = 2.21$	$\frac{b}{a} = 5.32$	$\frac{b}{a} = 7.09$	$\frac{b}{a} = 10.64$	$\frac{b}{a} = 25.11$
0.105	--	--	--	--	--
0.209	--	--	--	--	--
0.314	--	--	--	--	--
0.419	--	--	--	--	--
0.524	3.25	3.13	3.36	3.71	3.28
0.628	4.21	4.47	4.66	5.21	6.30
0.732	6.92	6.17	7.07	8.18	9.71
0.838	8.69	8.70	8.95	10.48	11.76
0.942	11.70	11.83	11.89	13.52	14.17
1.047	15.19	15.41	15.08	16.21	17.16
1.152	18.38	19.13	18.97	19.96	20.92
1.256	22.80	23.66	24.00	24.46	25.34
1.361	28.98	29.10	28.54	28.93	30.36
1.413	32.64	32.69	31.79	32.69	--
1.466	36.30	36.03	35.74	36.15	37.06
1.518	41.34	39.90	39.67	39.84	--
1.571	45.62	43.13	42.22	45.12	44.29
1.623	52.55	--	--	--	--
1.675	57.55	55.67	54.96	54.33	54.12
1.780	73.72	71.37	69.83	68.91	66.11
1.885	92.73	90.09	86.38	83.76	80.54
1.989	127.1	114.9	108.3	104.7	110.7
2.094	167.5	151.0	140.1	133.5	128.9
2.199	217.5	191.9	178.1	168.0	161.0
2.303	298.2	257.7	227.0	217.2	211.5
2.408	385.0	331.4	302.8	286.9	275.3
2.513	493.7	436.0	395.6	381.5	363.6
2.618	568.9	534.1	493.8	475.9	469.4
2.670	580.1	557.6	539.4	--	--
2.722	592.1	587.8	584.4	576.5	599.5
2.774	--	601.9	602.3	608.3	--
2.827	530.9	600.3	610.5	622.3	686.0
2.880	--	--	--	614.7	690.8
2.932	432.8	505.5	560.1	596.7	686.4
3.036	318.5	394.0	453.8	501.0	582.6
3.141	253.8	311.3	360.7	401.3	464.7
3.246	--	231.8	270.2	298.8	336.0
3.351	--	178.4	--	220.8	244.3
3.456	--	--	--	--	181.1

Table XVI (cont. -1)

λ = 60 cm.

$\frac{a}{\lambda}$ = 2.98 x 10^{-3}

REACTANCES					
$\beta_o h$	$\frac{b}{a}$ = 2.21	$\frac{b}{a}$ = 5.32	$\frac{b}{a}$ = 7.09	$\frac{b}{a}$ = 10.64	$\frac{b}{a}$ = 25.11
0.105	-808.8	-1104	-1141	-1311	-875.4
0.209	-507.5	- 609.4	- 637.2	- 757.8	-688.1
0.314	-380.7	- 431.1	- 449.5	- 501.8	-500.1
0.419	-304.0	- 339.8	- 348.9	- 377.0	-382.6
0.524	-248.7	- 271.8	- 282.9	- 297.7	-299.1
0.628	-211.8	- 228.1	- 234.0	- 237.8	-246.5
0.732	-188.4	- 189.1	- 189.7	- 194.0	-200.4
0.838	-146.1	- 154.5	- 157.7	- 161.6	-166.0
0.942	-118.1	- 123.8	- 124.3	- 127.7	-128.9
1.047	- 93.61	- 97.78	- 98.27	- 99.68	-100.2
1.152	- 69.36	- 72.06	- 73.36	- 73.24	- 73.10
1.256	- 46.49	- 48.47	- 49.06	- 49.08	- 47.73
1.361	- 23.44	- 25.27	- 24.29	- 24.22	- 22.90
1.413	- 11.09	- 11.86	- 11.65	- 10.69	--
1.466	0.24	0.31	0.59	0.72	2.27
1.518	11.97	11.17	11.84	11.66	--
1.571	21.63	21.37	21.28	23.12	24.95
1.623	34.79	--	--	--	--
1.675	48.27	47.96	47.43	48.22	51.60
1.780	73.29	73.73	73.48	74.07	78.07
1.885	101.3	99.53	98.53	99.57	103.6
1.989	135.4	128.3	125.0	127.0	131.7
2.094	165.5	154.7	152.0	157.9	161.3
2.199	186.3	182.0	178.6	186.6	188.8
2.303	208.5	203.7	207.6	213.5	221.1
2.408	191.1	211.8	220.0	223.5	248.1
2.513	147.1	192.9	212.6	230.2	263.0
2.618	42.89	110.9	166.3	187.9	238.5
2.670	- 36.71	51.96	126.3	--	--
2.722	-104.8	- 19.41	51.63	87.01	174.4
2.774	--	- 95.98	- 18.75	27.52	--
2.827	-243.0	- 171.5	- 96.62	- 49.28	32.78
2.880	--	--	--	- 129.5	- 42.80
2.932	-312.2	- 284.0	- 234.4	- 192.4	-153.1
3.036	-349.6	- 331.0	- 323.1	- 311.0	-298.1
3.141	-340.4	- 347.1	- 350.7	- 359.5	-377.3
3.246	--	- 331.1	- 348.5	- 359.6	-391.1
3.351	--	- 311.4	--	- 338.3	-371.5
3.456	--	--	--	--	-334.2

Table XVI (cont. -2)

$\lambda = 60$ cm.

$\frac{a}{\lambda} = 3.97 \times 10^{-3}$

RESISTANCES					
	$\frac{a}{c} = 1.33$				
$\beta_o h$	$\frac{b}{a} = 1.67$	$\frac{b}{a} = 4.00$	$\frac{b}{a} = 5.33$	$\frac{b}{a} = 8.00$	$\frac{b}{a} = 18.33$
0.105	--	--	--	--	--
0.209	--	--	--	--	--
0.314	--	--	--	--	--
0.419	2.13	1.85	1.61	1.75	4.05
0.524	3.14	2.48	3.98	2.83	6.06
0.628	4.45	3.45	5.46	4.84	7.76
0.732	5.98	6.15	7.64	6.89	9.60
0.838	7.85	8.65	9.29	9.91	11.58
0.942	11.08	11.50	12.20	12.85	14.27
1.047	14.38	15.01	15.46	18.38	17.51
1.152	18.32	18.56	19.42	20.09	20.44
1.256	23.52	23.65	24.31	24.41	25.62
1.361	29.98	29.49	29.33	29.47	30.57
1.413	33.56	33.20	32.83	33.47	--
1.466	37.04	37.05	36.72	36.58	37.68
1.518	42.22	40.51	39.85	40.06	--
1.571	48.21	44.78	45.08	44.36	45.54
1.675	60.33	58.42	56.81	55.03	56.07
1.780	80.24	74.41	71.24	69.25	67.18
1.885	103.5	92.75	89.81	86.65	83.23
1.989	137.1	120.0	113.5	109.4	103.2
2.094	182.4	154.3	145.9	138.1	130.4
2.199	238.7	201.4	188.9	176.4	166.2
2.303	306.1	263.1	241.5	227.2	211.5
2.408	393.7	336.7	312.5	296.9	274.5
2.513	461.4	428.1	397.5	389.6	353.5
2.566	475.2	470.0	438.6	--	--
2.618	488.7	498.1	483.1	462.8	455.8
2.670	481.6	521.1	512.8	528.8	--
2.722	465.5	531.6	536.9	--	544.1
2.774	--	--	--	555.5	582.6
2.827	392.5	507.1	522.3	554.1	596.9
2.880	--	--	--	538.1	--
2.932	319.6	436.0	464.0	501.2	579.4
3.036	247.6	345.0	375.6	419.3	504.0
3.141	191.9	267.6	292.8	325.1	395.9
3.246	--	199.4	222.5	250.2	295.3
3.351	--	--	--	191.4	217.5
3.456	--	--	--	--	163.0

Table XVI (cont. -3)

λ = 60 cm.

$\frac{a}{\lambda} = 3.97 \times 10^{-3}$

REACTANCES					
	$\frac{a}{c} = 1.33$				
$\beta_o h$	$\frac{b}{a} = 1.67$	$\frac{b}{a} = 4.00$	$\frac{b}{a} = 5.33$	$\frac{b}{a} = 8.00$	$\frac{b}{a} = 18.33$
0.105	-694.0	-980.0	-1006	-1100	-957.8
0.209	-443.1	-522.9	- 572.2	- 632.5	-610.8
0.314	-333.5	-376.6	- 398.8	- 427.6	-445.3
0.419	-269.8	-297.2	- 301.3	- 309.2	-342.8
0.524	-221.9	-241.0	- 250.7	- 259.2	-271.3
0.628	-184.9	-199.0	- 205.9	- 212.3	-221.0
0.732	-156.1	-165.9	- 169.6	- 177.1	-181.1
0.838	-130.0	-136.2	- 138.5	- 143.6	-146.3
0.942	-105.6	-110.2	- 111.4	- 114.8	-116.9
1.047	- 83.81	- 86.62	- 87.32	- 89.62	- 90.35
1.152	- 62.81	- 64.59	- 64.73	- 65.98	- 60.73
1.256	- 42.00	- 42.53	- 42.37	- 43.42	- 42.61
1.361	- 20.50	- 20.49	- 20.64	- 20.49	- 19.38
1.413	- 9.93	- 9.09	- 8.76	- 8.49	--
1.466	0.48	1.01	1.25	2.09	3.57
1.518	11.62	12.48	12.16	12.99	--
1.571	21.75	23.71	22.85	24.26	26.12
1.675	43.35	45.63	45.31	46.16	48.96
1.780	68.75	70.54	73.19	69.52	71.96
1.885	93.52	93.38	92.03	93.81	97.30
1.989	116.7	116.9	116.3	117.9	121.7
2.094	138.9	141.7	142.1	144.9	149.1
2.199	148.4	162.7	165.4	168.4	175.4
2.303	142.0	175.7	182.2	185.4	196.0
2.408	108.6	170.9	188.4	192.9	215.7
2.513	28.19	136.4	161.1	192.9	220.2
2.566	- 24.52	101.1	128.6	--	--
2.618	- 75.54	60.06	90.60	140.3	194.3
2.670	-138.0	- 0.69	40.85	44.64	--
2.722	-183.0	- 59.50	- 18.85	--	115.4
2.774	--	--	--	- 19.34	55.90
2.827	-261.6	-174.7	- 143.6	- 89.45	- 19.23
2.880	--	--	--	- 151.9	--
2.932	-295.5	-261.7	- 246.3	- 216.9	-168.8
3.036	-297.6	-304.1	- 299.7	- 287.0	-280.5
3.141	-291.2	-313.3	- 313.6	- 322.9	-335.0
3.246	--	-304.7	- 308.7	- 317.1	-345.4
3.351	--	--	--	- 298.7	-324.9
3.456	--	--	--	--	-294.4

Table XVI (cont. -4)

$\lambda = 60$ cm.

$\frac{a}{\lambda} = 9.26 \times 10^{-3}$

RESISTANCES				
$\beta_o h$	$\frac{b}{a} = 1.71$	$\frac{b}{a} = 2.28$	$\frac{b}{a} = 3.43$	$\frac{b}{a} = 8.10$
0.105	--	--	--	--
0.209	1.00	0.59	1.55	--
0.314	1.14	1.43	2.02	--
0.419	2.53	2.46	2.70	2.82
0.524	3.57	2.81	3.62	5.63
0.628	4.50	4.12	5.50	7.47
0.732	6.31	6.42	7.34	9.64
0.838	8.44	8.64	9.87	11.94
0.942	11.00	11.58	12.72	14.42
1.047	13.63	15.04	16.28	17.37
1.152	18.28	19.24	20.34	21.62
1.256	23.02	24.46	23.67	26.42
1.361	29.92	30.79	30.12	31.57
1.466	38.55	39.08	38.46	38.80
1.571	48.56	49.39	49.03	48.01
1.675	63.83	62.88	59.88	58.21
1.780	84.20	81.53	75.32	72.12
1.885	106.5	104.9	96.27	89.49
1.989	137.6	134.4	124.8	107.0
2.094	175.2	171.7	157.4	135.1
2.199	220.8	214.1	199.0	168.9
2.303	263.2	256.2	238.8	212.9
2.408	293.1	294.9	287.9	260.2
2.513	289.5	316.0	318.8	306.7
2.618	274.6	305.2	330.2	353.5
2.722	236.1	273.8	315.3	381.7
2.827	198.6	235.5	282.6	373.7
2.932	164.7	195.4	235.6	336.3
3.036	138.1	159.7	195.0	285.7
3.141	110.1	127.0	160.0	232.1

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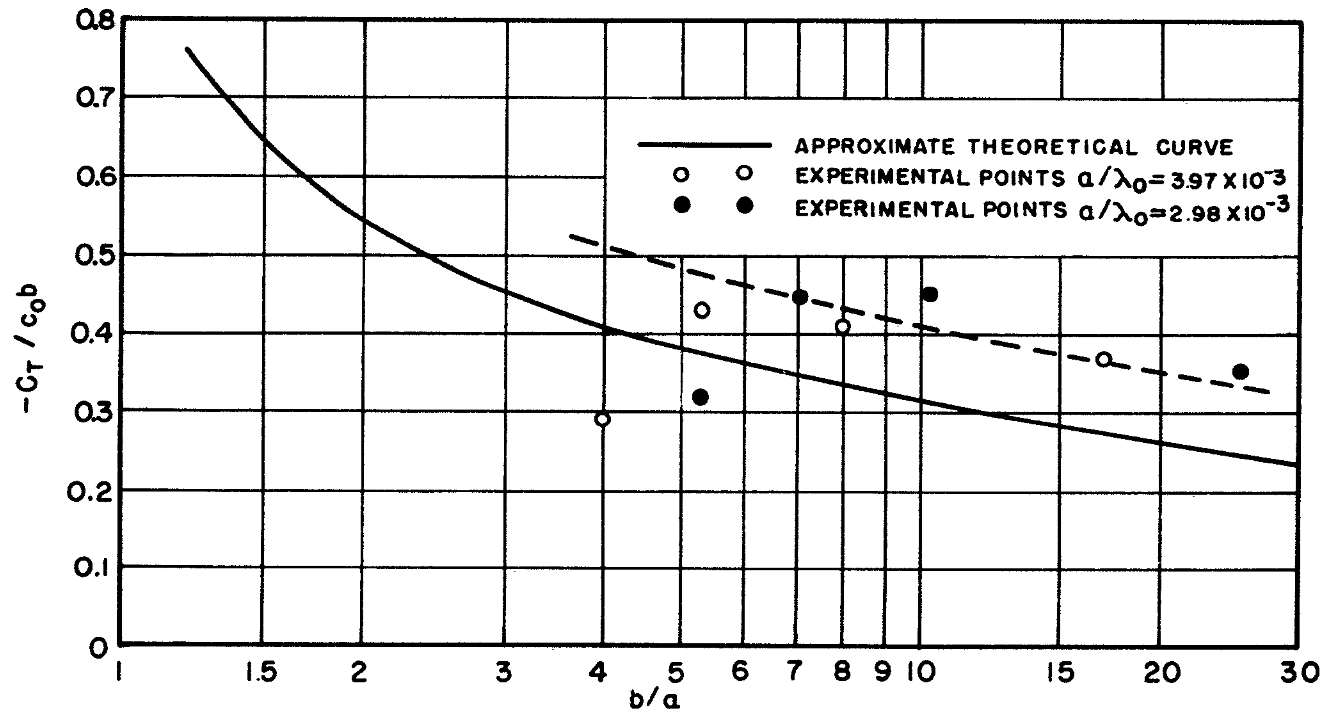


FIG. 49 CAPACITIVE END-EFFECT CORRECTION FOR ANTENNA DRIVEN FROM COAXIAL LINE (HARTIG)

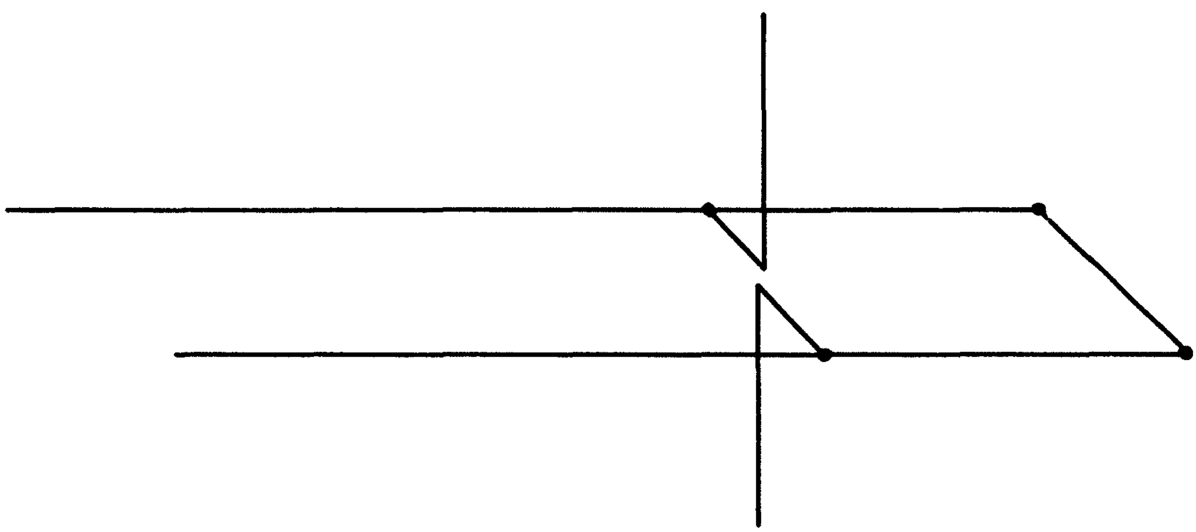


FIG. 48 CYLINDRICAL, STUB-SUPPORTED END-LOAD ANTENNA MOUNTED PERPENDICULAR TO THE PLANE OF THE LINE

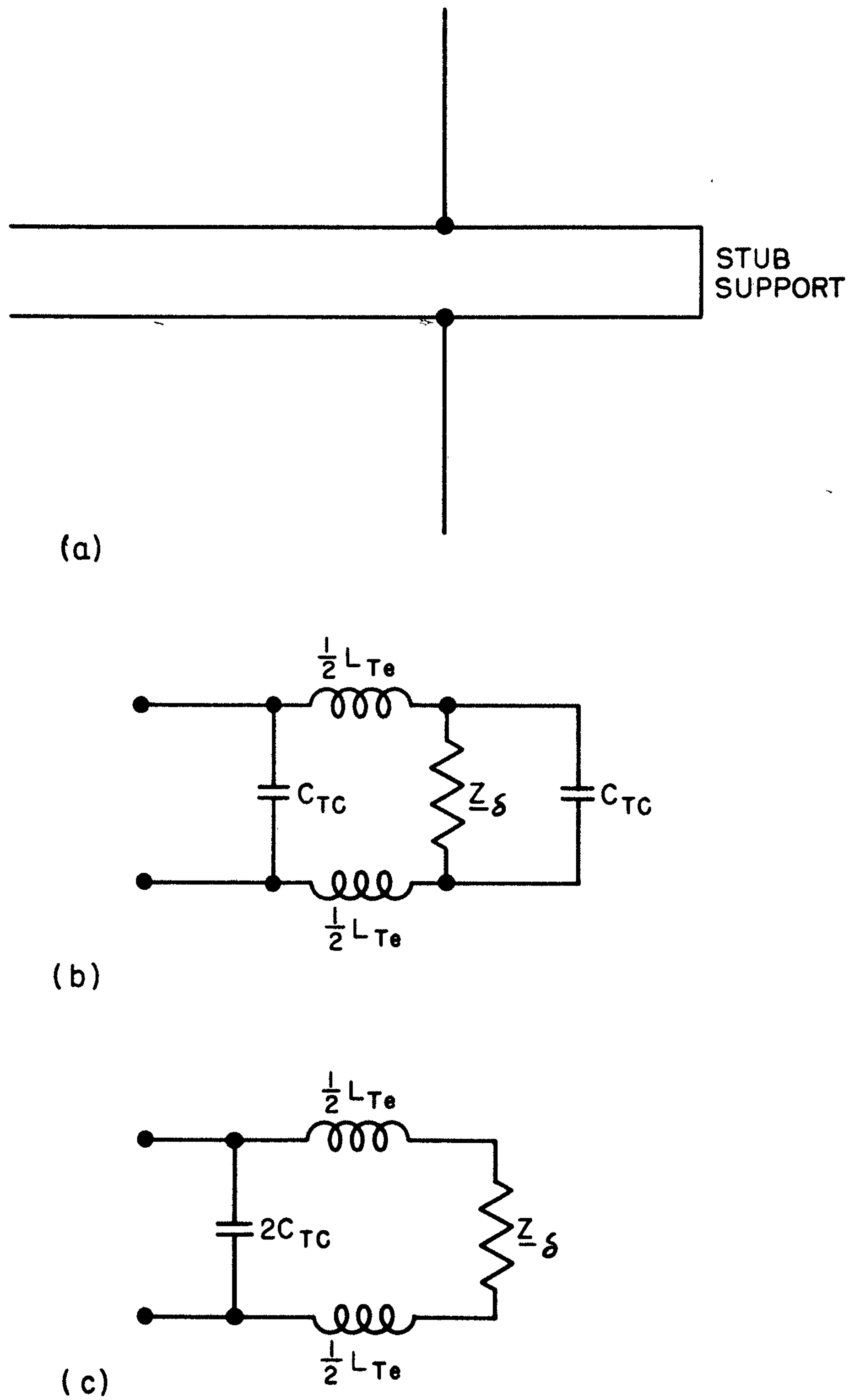


FIG. 47 STUB-SUPPORTED ANTENNA IN THE PLANE OF THE LINE

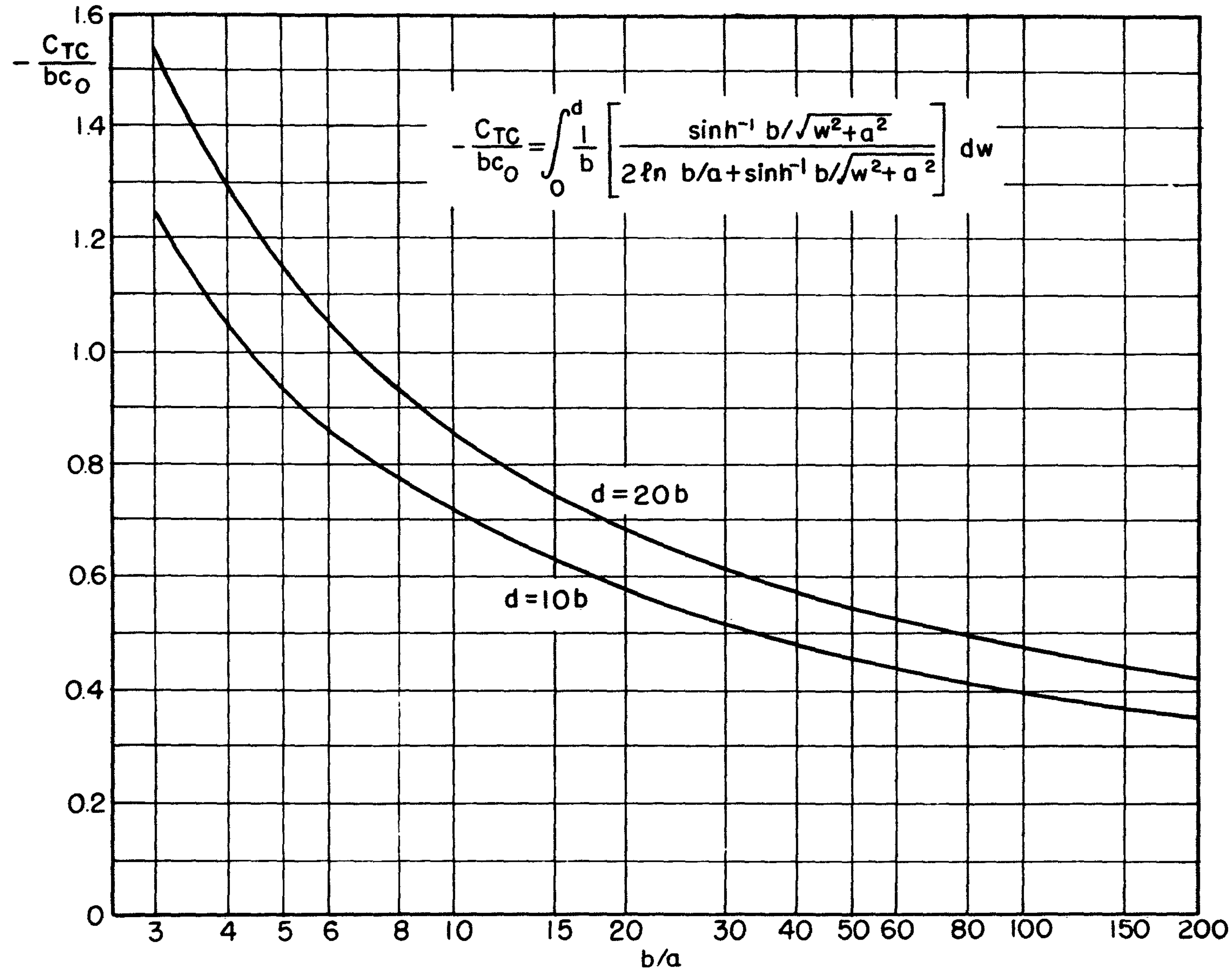


FIG. 46 CAPACITIVE END-EFFECT CORRECTION FOR ANTENNA AS A CENTER LOAD OR AS END LOAD WITH STUB SUPPORT

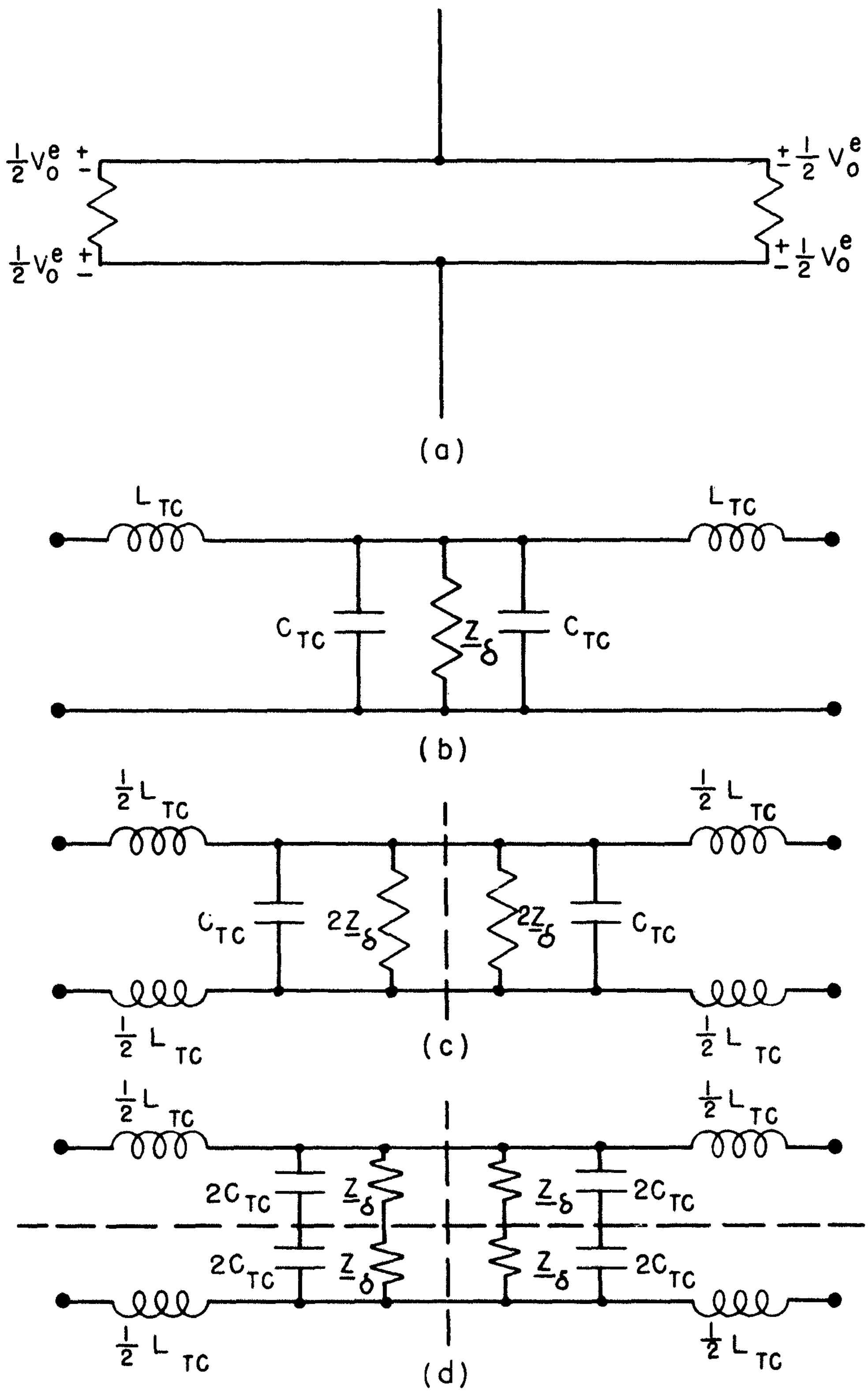


FIG. 45 CYLINDRICAL, CENTER DRIVEN ANTENNA AS A CENTER LOAD

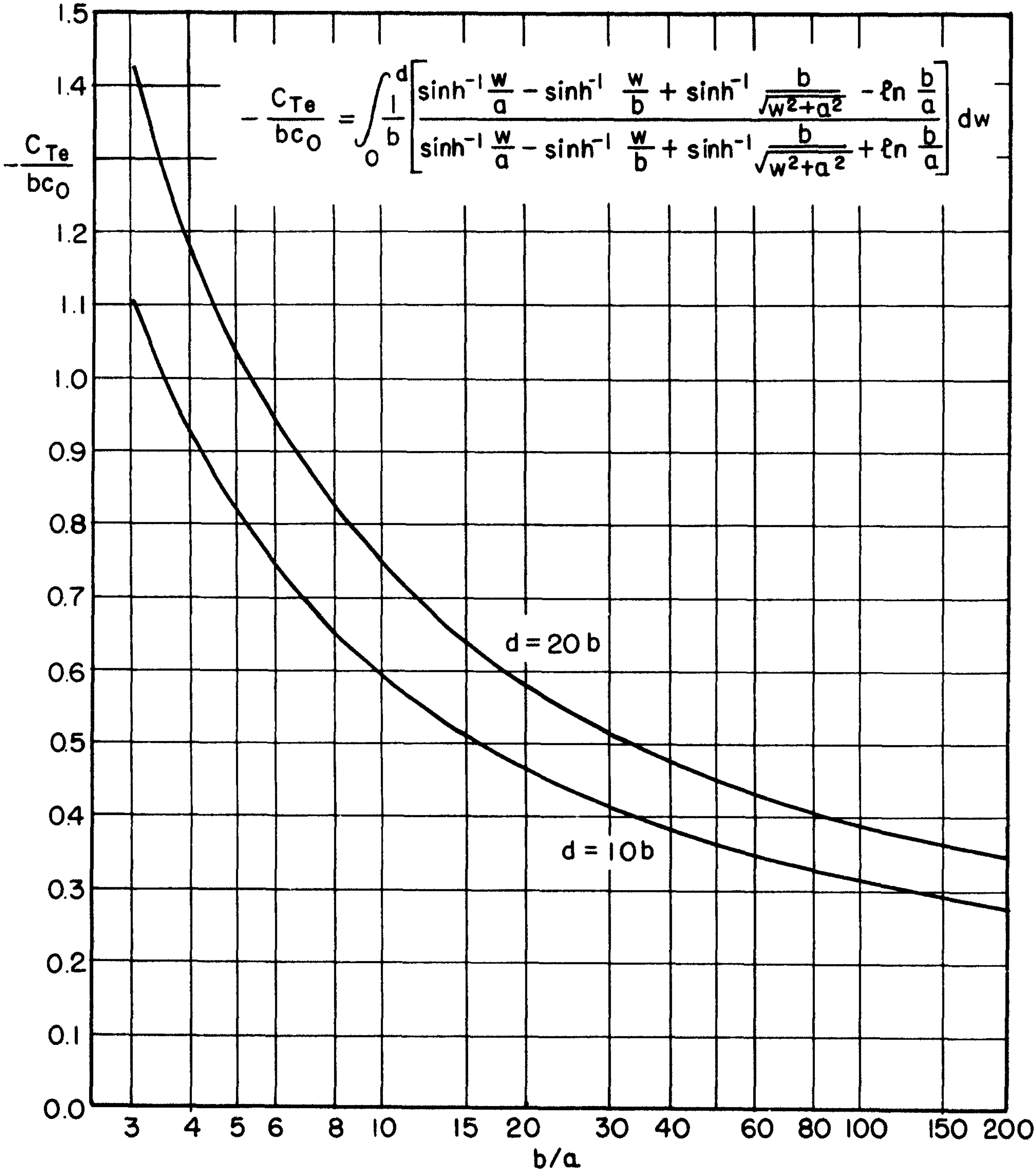
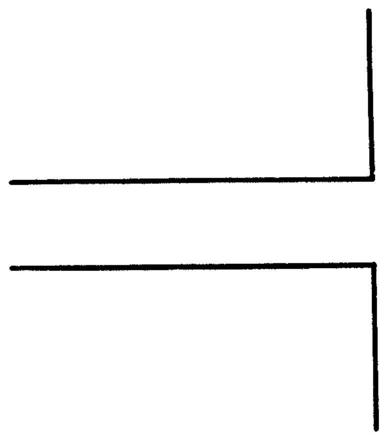
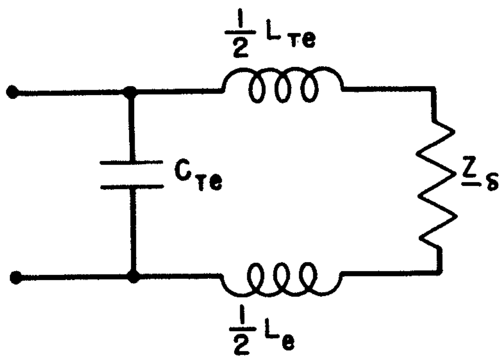


FIG. 44 CAPACITIVE END-EFFECT CORRECTION FOR ANTENNA AS END LOAD ON TWO-WIRE LINE



(a)



(b)

FIG. 43 CYLINDRICAL, CENTER-DRIVEN ANTENNA AS AN END LOAD

Table XVI (cont. -5)

λ = 60 cm.

$\frac{a}{\lambda} = 9.26 \times 10^{-3}$

REACTANCES				
$\beta_o h$	$\frac{b}{a} = 1.71$	$\frac{b}{a} = 2.28$	$\frac{b}{a} = 3.43$	$\frac{b}{a} = 8.10$
0.105	-433.8	-482.4	-572.2	-635.2
0.209	-283.3	-307.9	-352.5	-414.3
0.314	-220.1	-237.5	-261.8	-304.3
0.419	-179.1	-191.8	-208.4	-237.4
0.524	-150.3	-159.0	-171.4	-192.2
0.628	-126.2	-133.9	-141.5	-157.5
0.732	-107.7	-112.4	-117.8	-129.0
0.838	- 90.06	- 93.54	- 98.36	-105.3
0.942	- 73.50	- 75.96	- 78.89	- 83.37
1.047	- 58.50	- 59.53	- 61.20	- 63.97
1.152	- 43.31	- 43.58	- 44.88	- 45.54
1.256	- 27.62	- 28.46	- 27.17	- 27.86
1.361	- 12.92	- 11.92	- 10.47	- 11.06
1.466	2.64	2.93	4.66	5.70
1.571	18.01	19.13	21.12	21.86
1.675	31.90	34.23	35.80	39.34
1.780	47.53	50.21	52.84	56.33
1.885	60.21	64.11	69.33	72.57
1.989	70.67	76.02	81.36	89.12
2.094	72.49	80.63	88.52	98.81
2.199	51.62	70.65	87.05	114.1
2.303	17.35	48.66	73.03	120.6
2.408	- 31.34	4.44	44.96	115.2
2.513	- 87.51	- 53.67	- 6.04	85.35
2.618	-134.6	-109.3	- 63.25	44.08
2.722	-175.2	-154.1	-123.8	- 28.61
2.827	-194.9	-186.9	-169.9	-101.0
2.932	-196.8	-203.3	-197.7	-166.9
3.036	-196.5	-201.7	-207.6	-200.4
3.141	-187.2	-197.1	-205.5	-223.6

Table XVI (Cont. -6)

$\lambda = 60$ cm

$\frac{a}{\lambda} = 1.59 \times 10^{-2}$

	RESISTANCES			REACTANCES		
$\beta_o h$	$\frac{b}{a}=1.33$	$\frac{b}{a}=2.00$	$\frac{b}{a}=4.72$	$\frac{b}{a}=1.33$	$\frac{b}{a}=2.00$	$\frac{b}{a}=4.72$
0.015	--	--	--	-247.6	-368.5	-469.0
0.209	--	--	0.89	-172.0	-232.3	-308.6
0.314	1.42	1.96	3.50	-139.6	-178.6	-229.7
0.419	1.71	2.79	4.84	-117.9	-144.8	-181.8
0.524	2.38	3.66	6.02	-101.1	-120.9	-146.4
0.628	3.60	4.97	7.23	- 35.48	-101.7	-119.5
0.732	5.13	6.99	9.58	- 75.42	- 85.48	- 98.46
0.838	7.40	9.83	11.57	- 64.31	- 70.93	- 79.93
0.942	9.87	11.91	14.32	- 53.40	- 57.96	- 63.32
1.047	13.37	15.65	17.71	- 42.51	- 45.25	- 48.35
1.152	17.48	19.30	21.57	- 31.63	- 33.14	- 34.09
1.256	22.60	24.32	26.37	- 21.22	- 21.98	- 19.11
1.361	30.66	30.47	32.34	- 10.49	- 10.23	- 6.88
1.466	39.64	39.06	39.16	0.04	0.37	5.86
1.571	52.67	52.19	49.02	10.00	11.90	18.49
1.675	68.96	65.93	60.39	17.06	20.42	31.39
1.780	88.22	85.86	73.07	22.74	30.08	42.69
1.885	110.9	104.9	89.65	23.50	34.24	52.78
1.989	137.5	129.4	110.1	8.46	35.97	61.66
2.094	156.8	157.2	134.3	- 17.07	29.80	66.31
2.199	170.5	184.8	161.9	- 44.95	10.57	68.60
2.303	170.6	203.5	198.8	- 73.11	- 18.40	61.45
2.408	159.8	211.6	229.5	- 99.44	- 51.56	40.87
2.513	144.0	206.3	254.1	-117.3	- 87.66	5.76
2.618	125.4	191.9	268.0	-129.6	-119.0	- 34.98
2.722	105.9	168.9	265.4	-133.9	-132.7	- 80.82
2.827	91.69	146.4	244.2	-133.0	-144.8	-121.0
2.932	78.09	122.3	213.0	-130.0	-147.4	-147.8
3.036	65.32	102.0	187.8	-125.7	-145.4	-160.4
3.141	56.76	83.13	149.2	-119.9	-139.0	-165.4

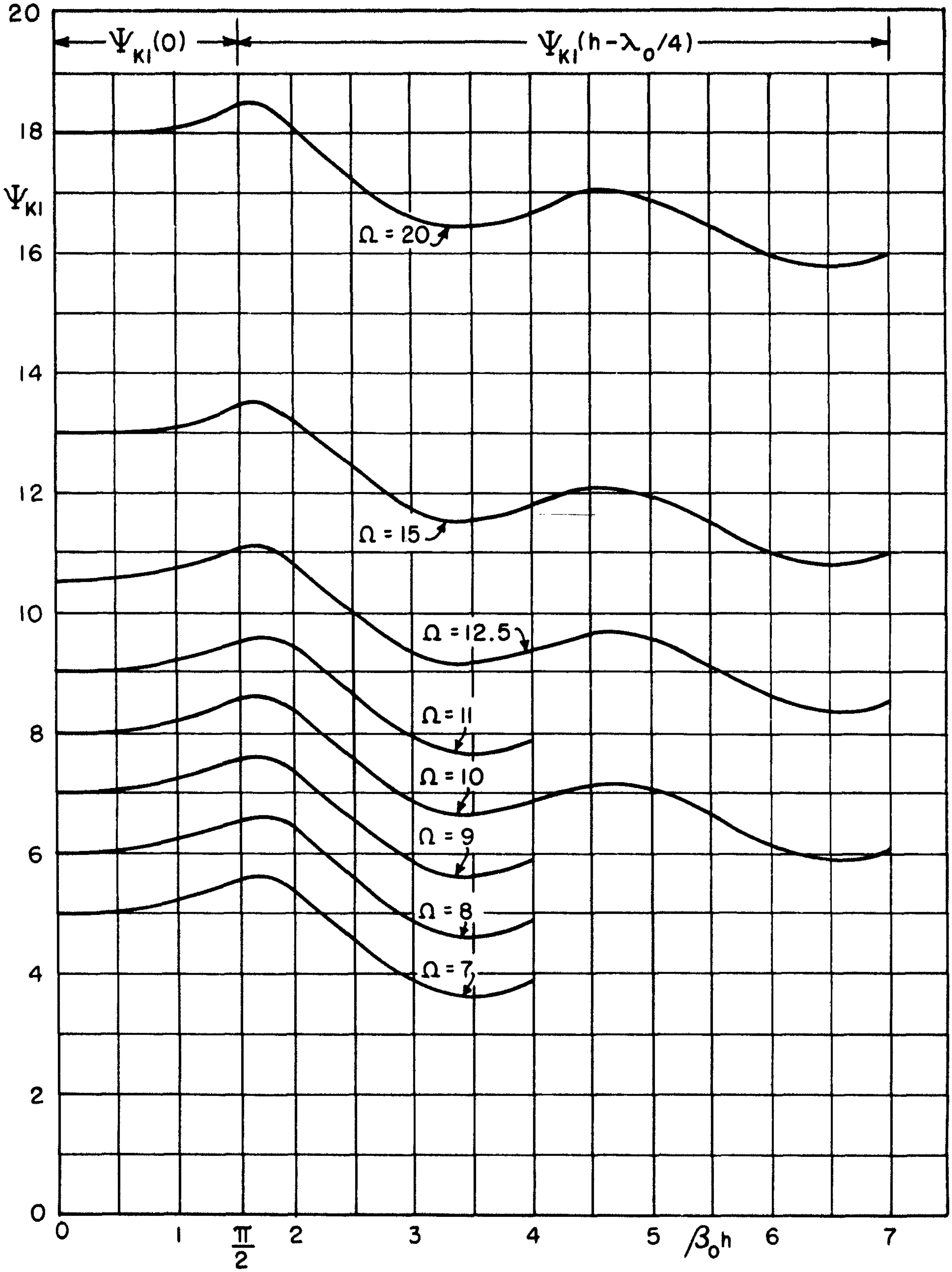


FIG. 1 EXPANSION PARAMETER Ψ_{K1}

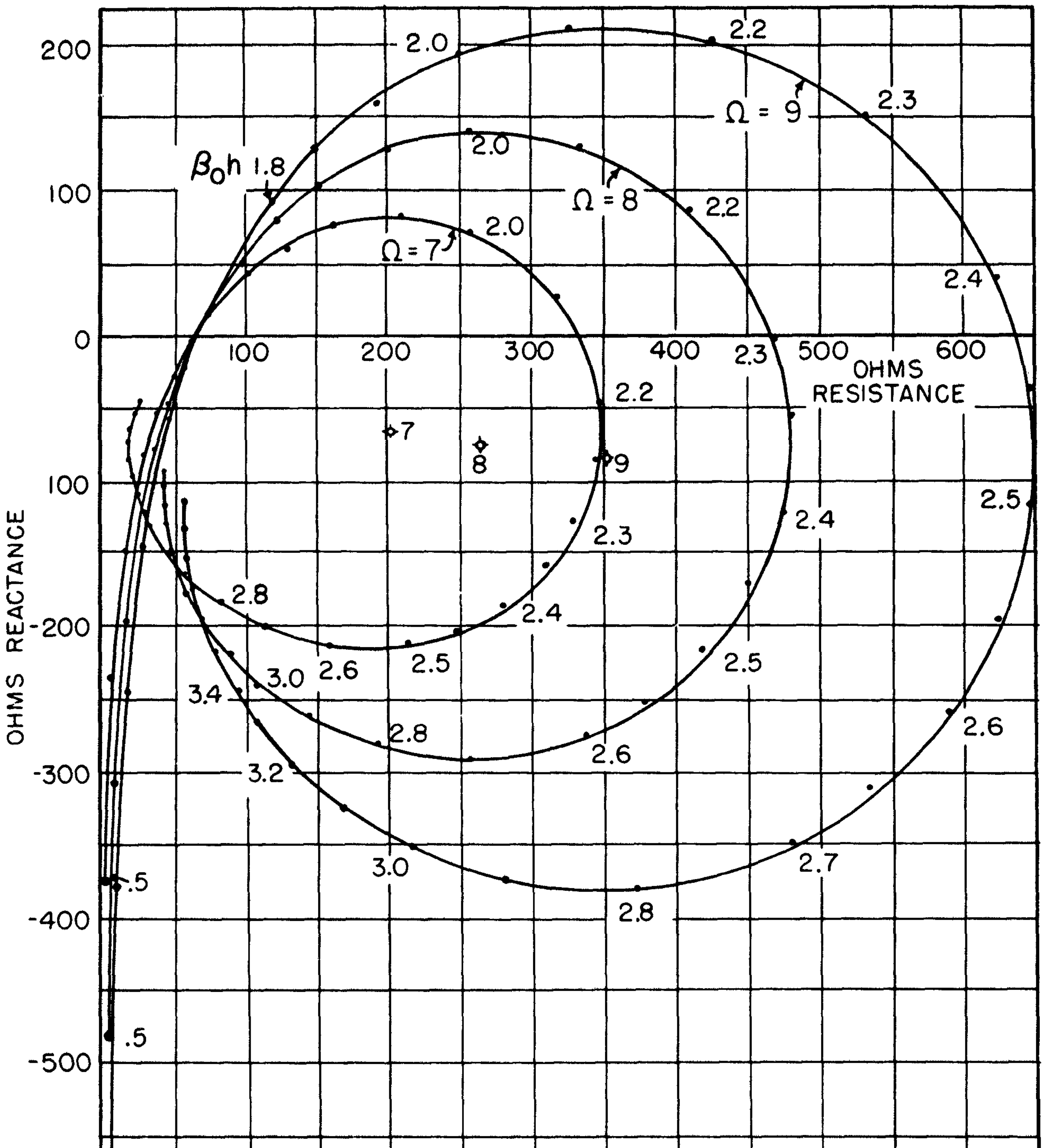


FIG. 2 KING-MIDDLETON SECOND-ORDER IMPEDANCE

$\Omega = 7, 8, 9$

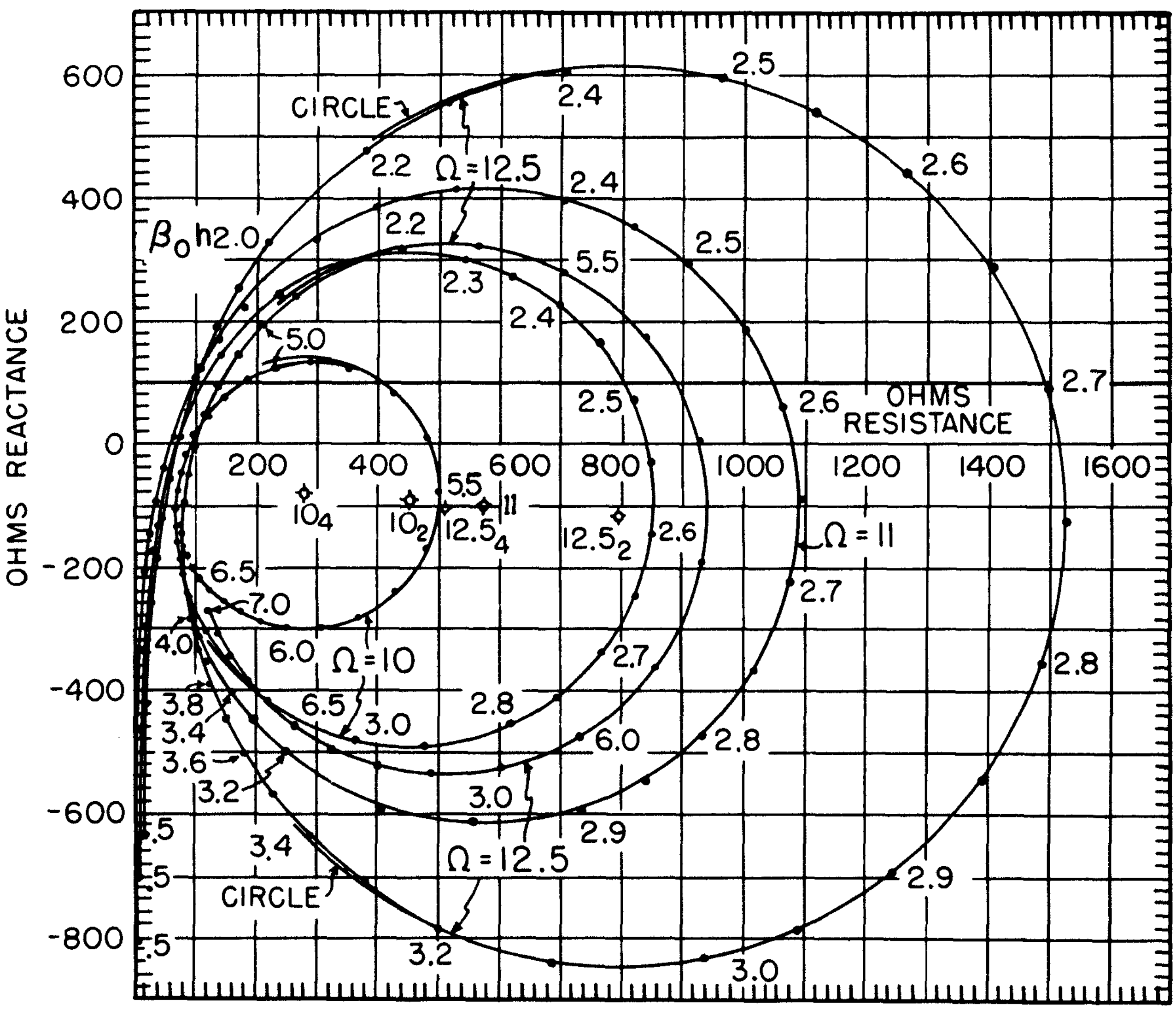


FIG. 3 KING-MIDDLETON SECOND-ORDER IMPEDANCE

$\Omega = 10, 11, 12.5$

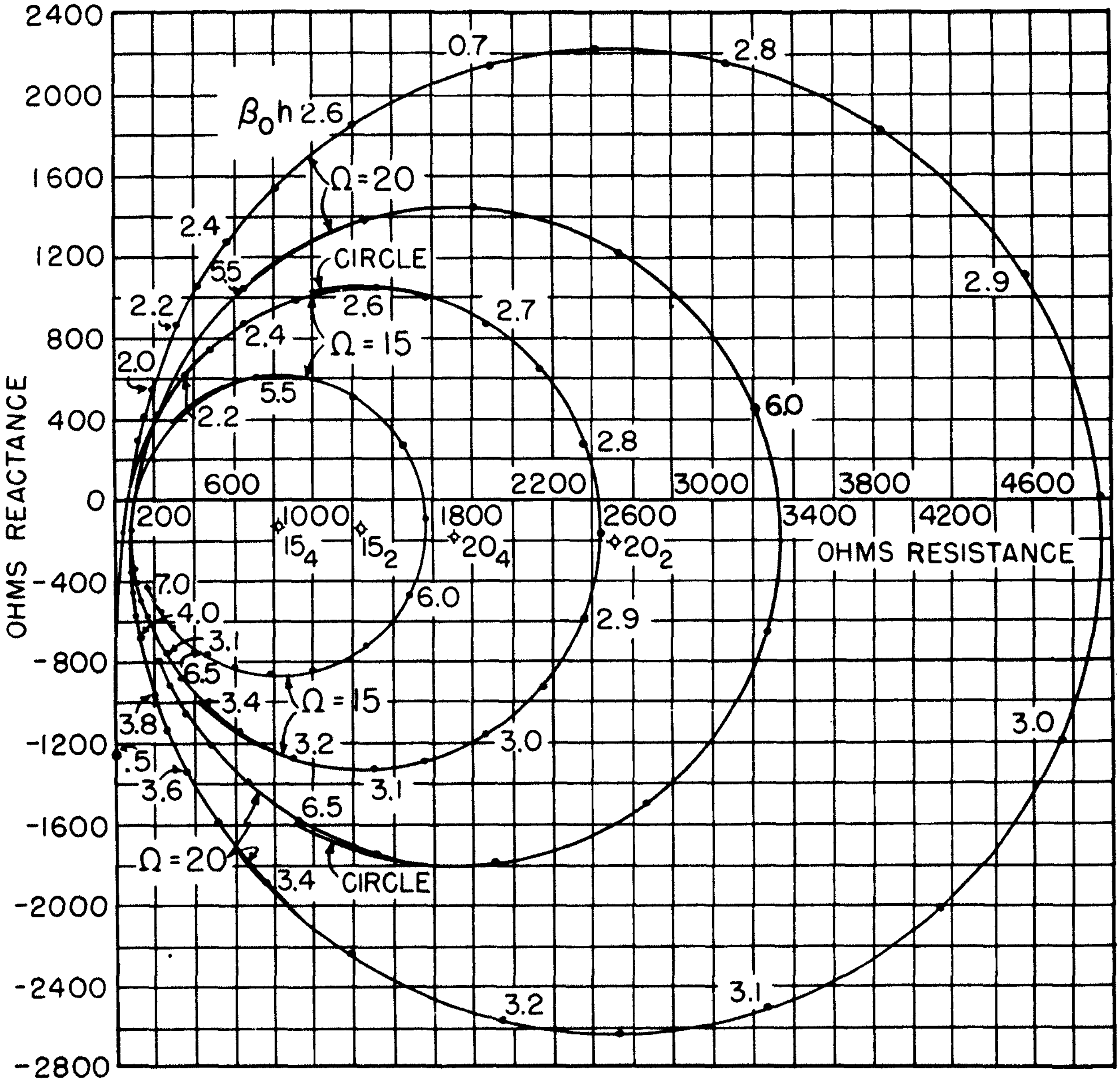
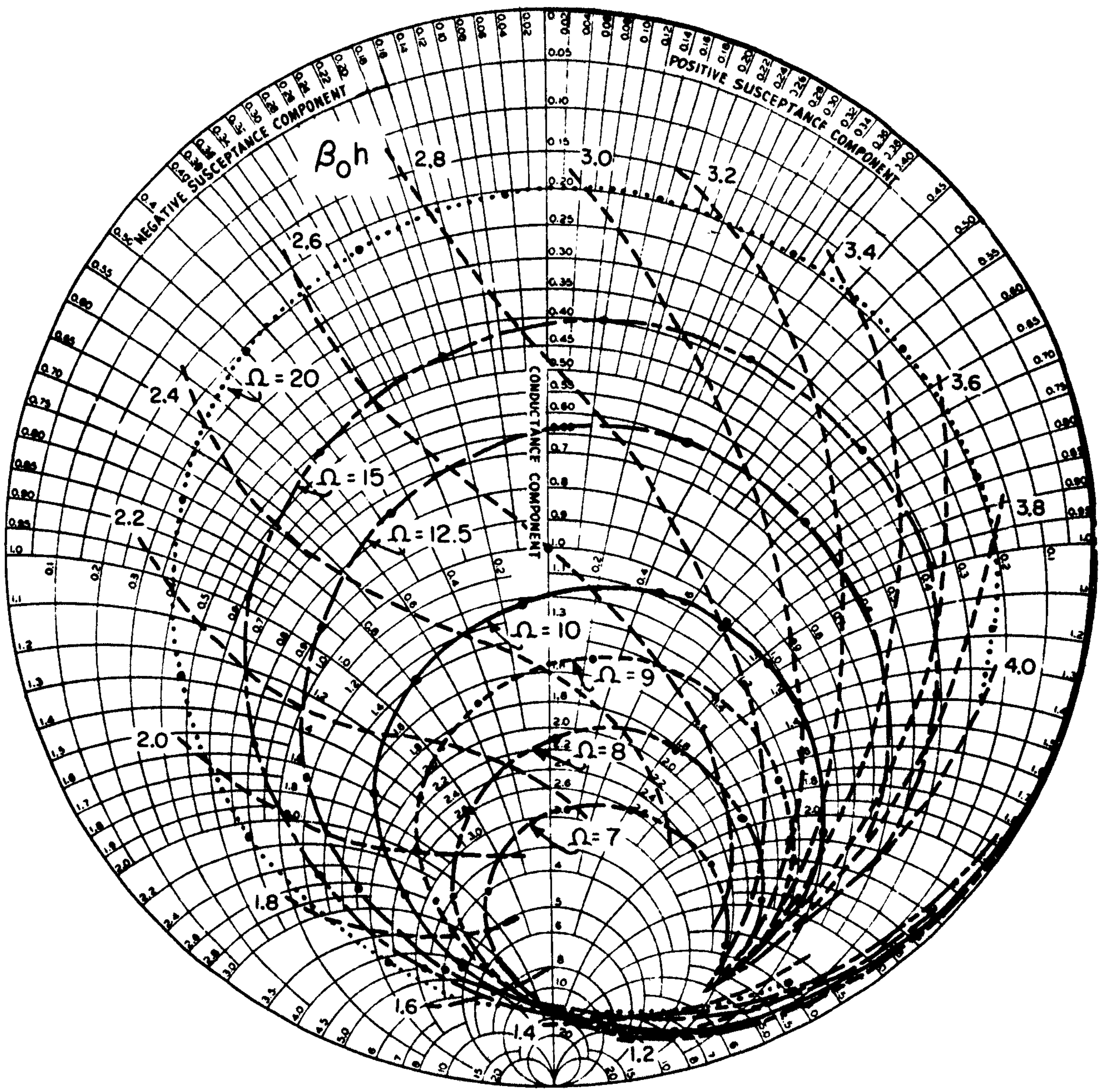


FIG. 4 KING-MIDDLETON SECOND-ORDER IMPEDANCE
Ω = 15,20

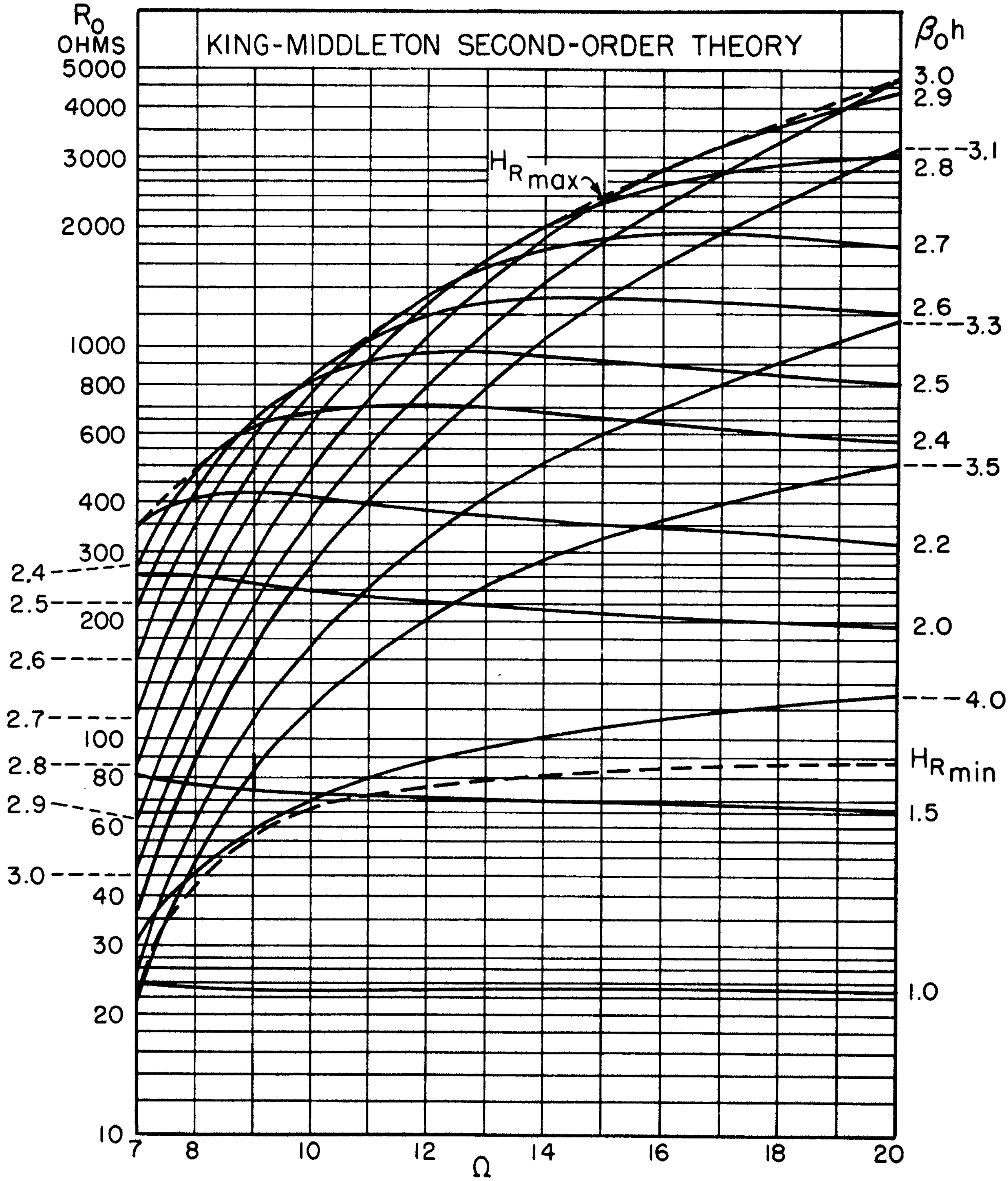


INPUT ADMITTANCE OF CENTER-DRIVEN DIPOLE

MHOS x 10^{-3} vs $\beta_0 h$

AFTER KING-MIDDLETON

FIGURE 5



KING-MIDDLETON SECOND-ORDER THEORY
R_0 OHMS
5000
4000
3000
2000
1000
800
600
400
300
200
100
80
60
40
30
20
10
2.4
2.5
2.6
2.7
2.8
2.9
3.0
$\beta_0 h$
3.0
2.9
3.1
2.8
2.7
2.6
3.3
2.5
2.4
3.5
2.2
2.0
4.0
$H_{R_{min}}$
1.5
1.0
$H_{R_{max}}$
7
8
10
12
14
16
18
20
Ω

FIGURE 6

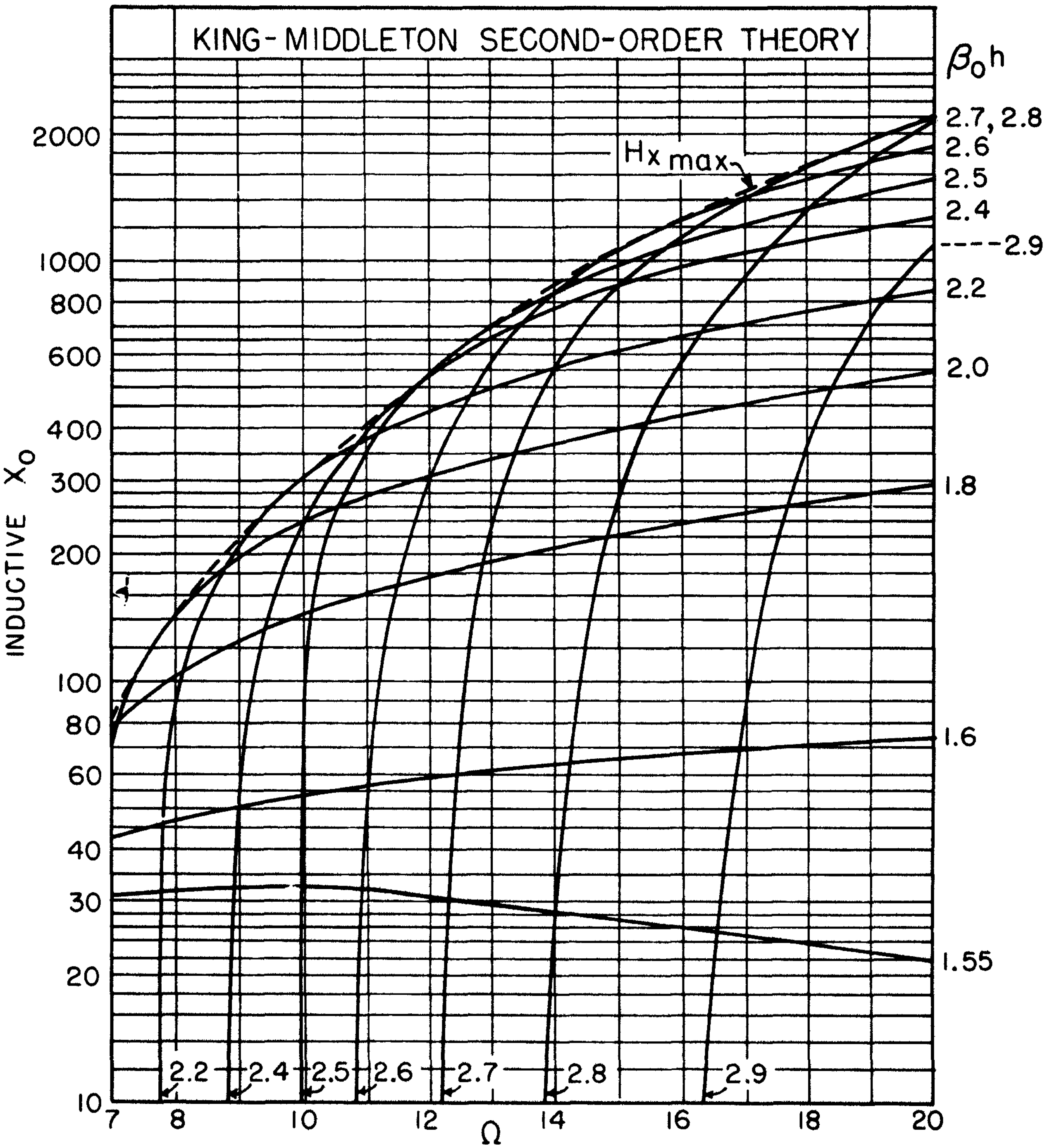

KING-MIDDLETON SECOND-ORDER THEORY
$\beta_0 h$
2.7, 2.8
2.6
2.5
2.4
2.9
2.2
2.0
1.8
1.6
1.55
H_x max
INDUCTIVE X_0
2000
1000
800
600
400
300
200
100
80
60
40
30
20
10
2.2
2.4
2.5
2.6
2.7
2.8
2.9
7
8
10
12
14
16
18
20
Ω

FIGURE 7a

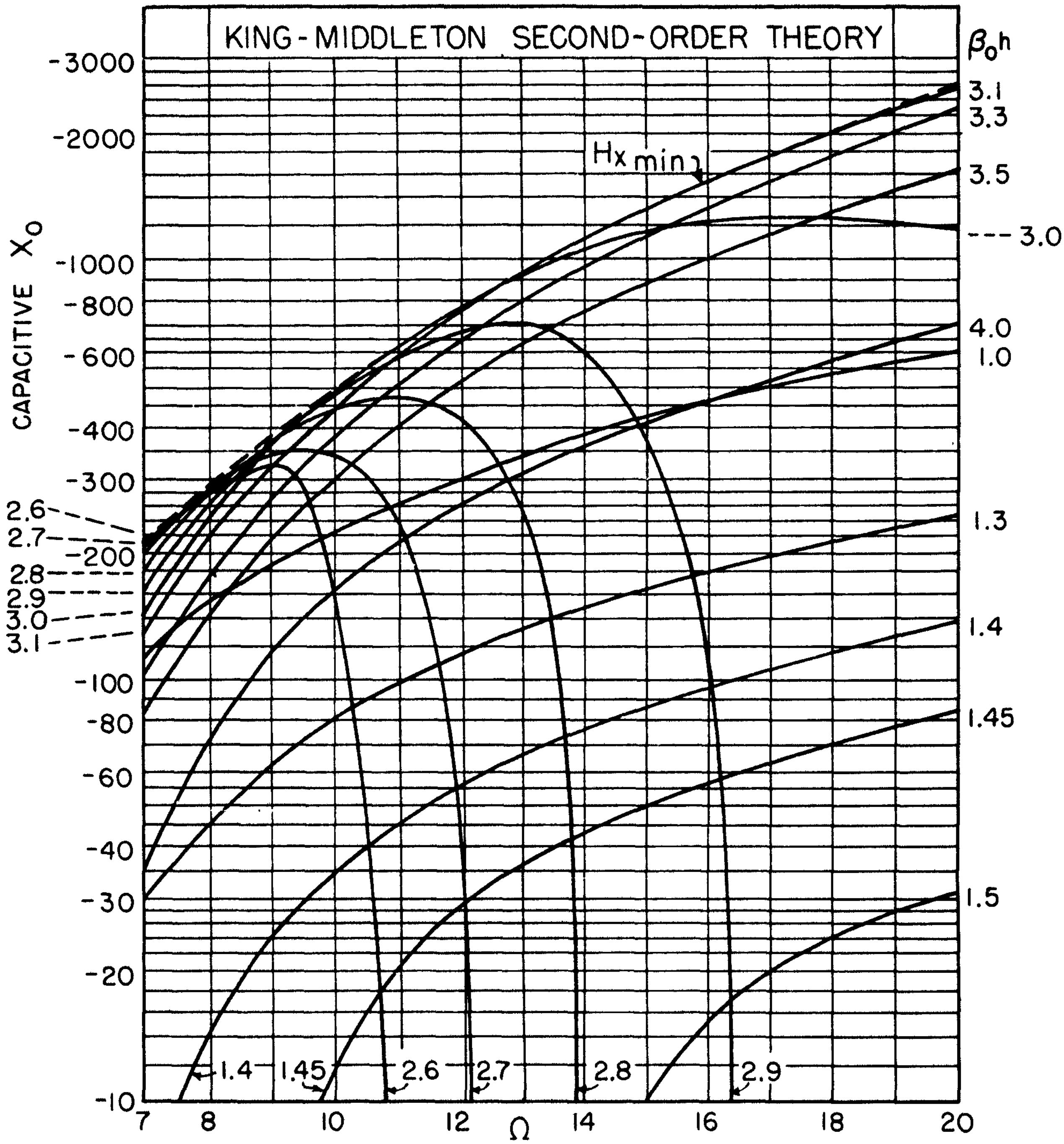
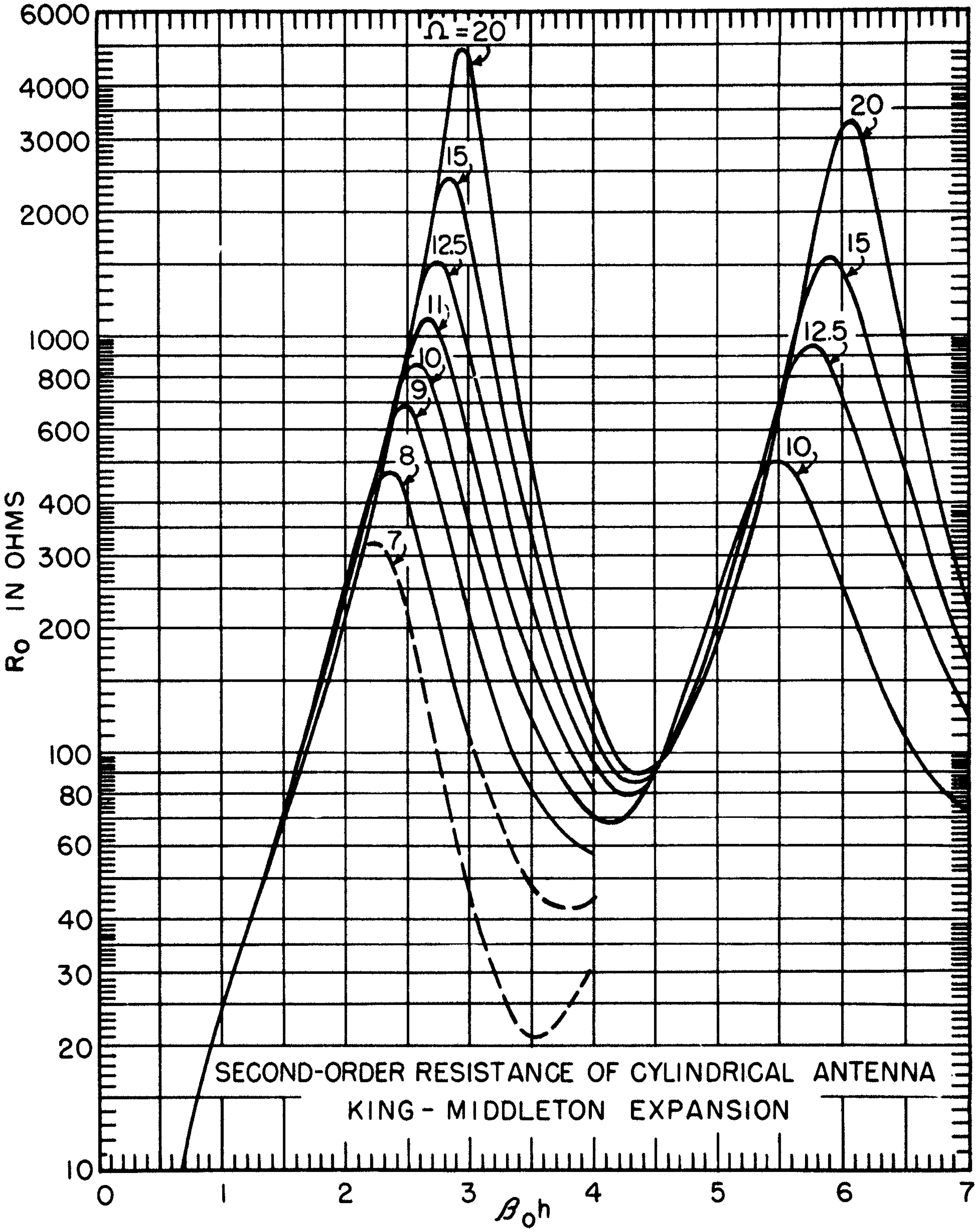

KING-MIDDLETON SECOND-ORDER THEORY
$\beta_0 h$
CAPACITIVE X_0
-3000
-2000
-1000
-800
-600
-400
-300
-200
-100
-80
-60
-40
-30
-20
-10
Hx min
3.1
3.3
3.5
3.0
4.0
1.0
1.3
1.4
1.45
1.5
2.6
2.7
2.8
2.9
3.0
3.1
1.4
1.45
2.6
2.7
2.8
2.9
7
8
10
12
14
16
18
20
Ω

FIGURE 7 b



6000
4000
3000
2000
1000
800
600
400
300
200
100
80
60
40
30
20
10
R_o IN OHMS
0
1
2
3
4
5
6
7
$\beta_o h$
$\Omega = 20$
15
12.5
11
10
9
8
7
20
15
12.5
10
SECOND-ORDER RESISTANCE OF CYLINDRICAL ANTENNA
KING - MIDDLETON EXPANSION

FIGURE 8

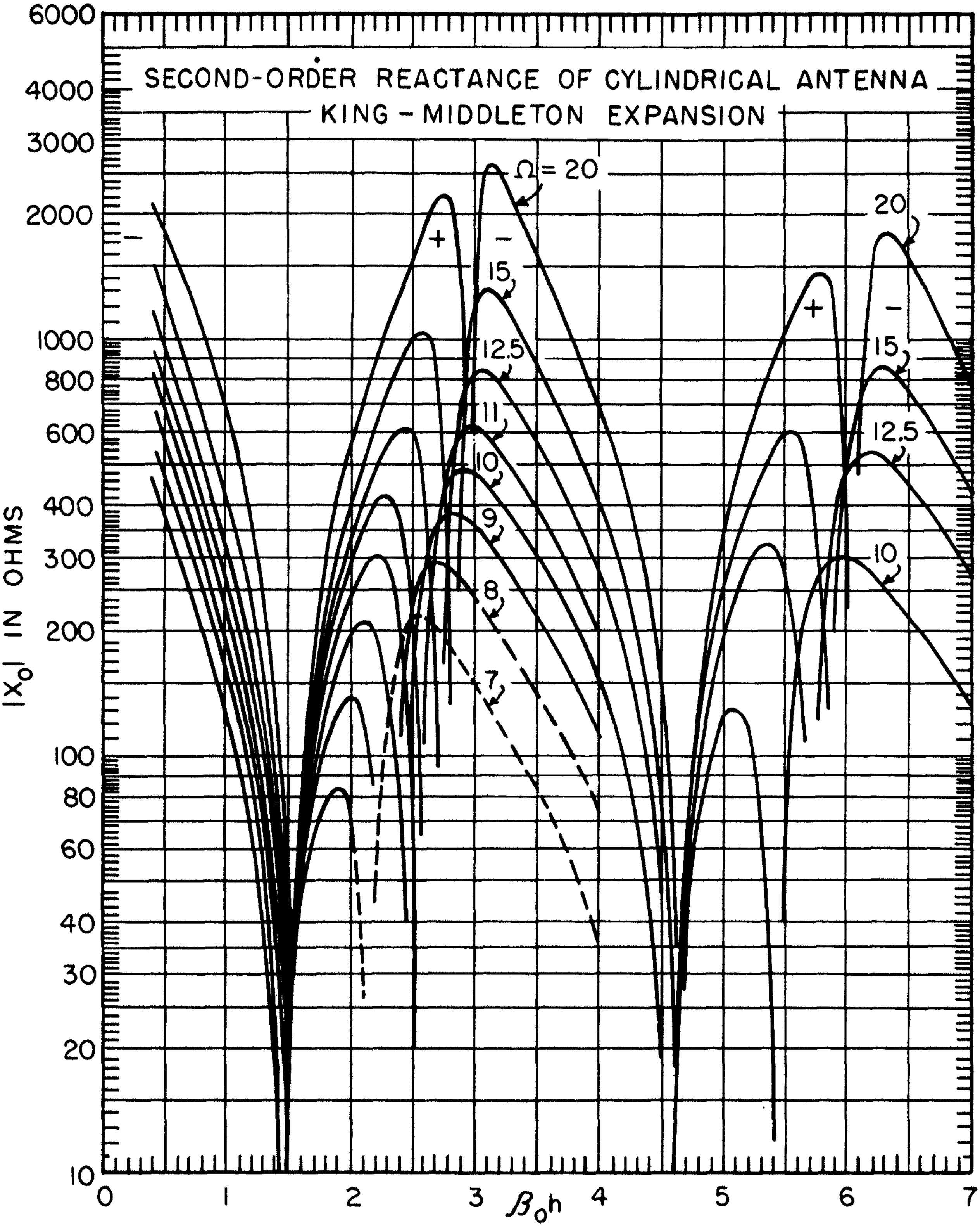

SECOND-ORDER REACTANCE OF CYLINDRICAL ANTENNA
KING - MIDDLETON EXPANSION
$\Omega = 20$
15
12.5
11
10
9
8
7
20
15
12.5
10
+
−
$|X_0|$ IN OHMS
$\beta_0 h$
6000
4000
3000
2000
1000
800
600
400
300
200
100
80
60
40
30
20
10
0
1
2
3
4
5
6
7

FIGURE 9

KING-MIDDLETON SECOND-ORDER REACTANCE

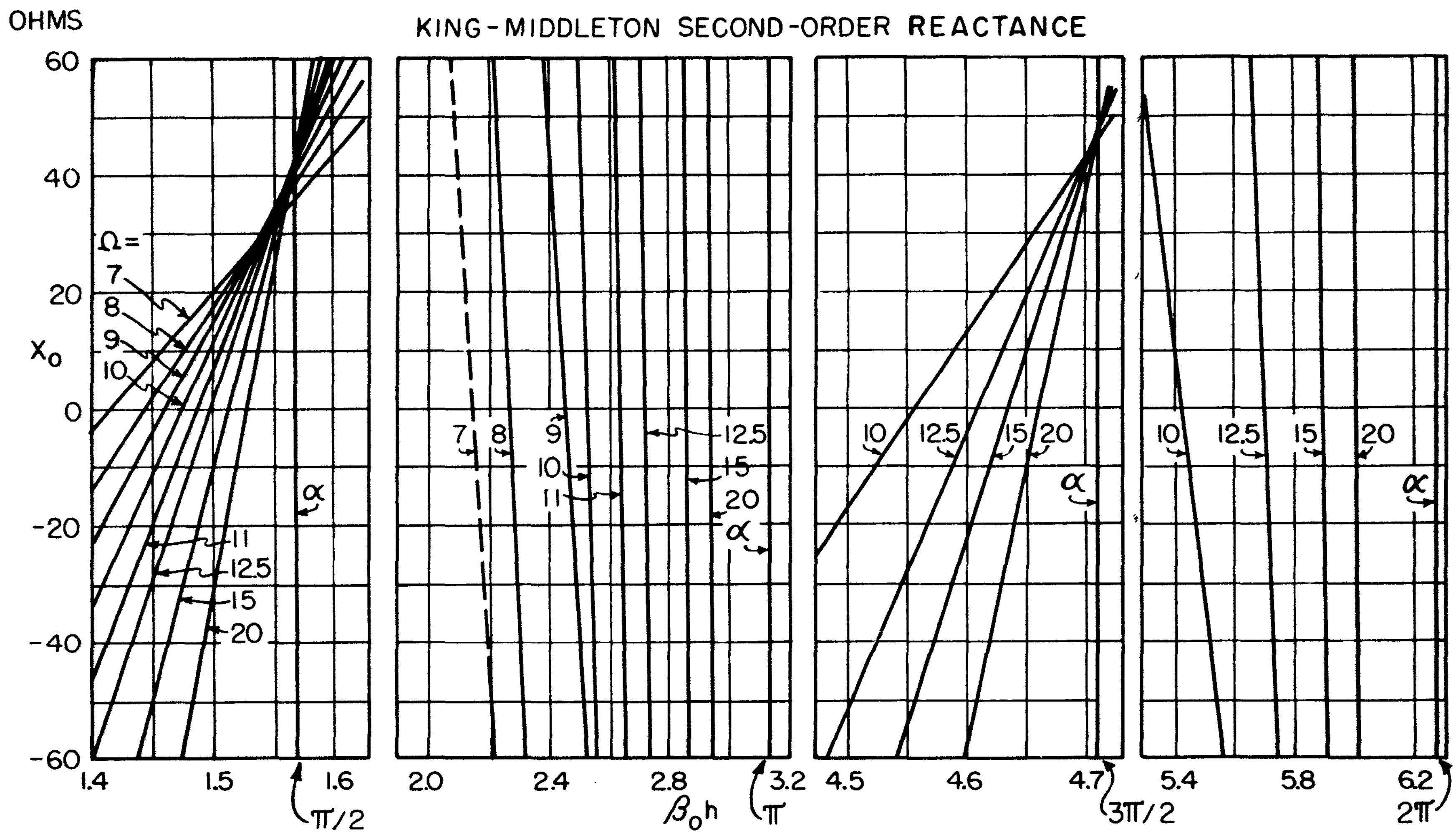


FIGURE 10

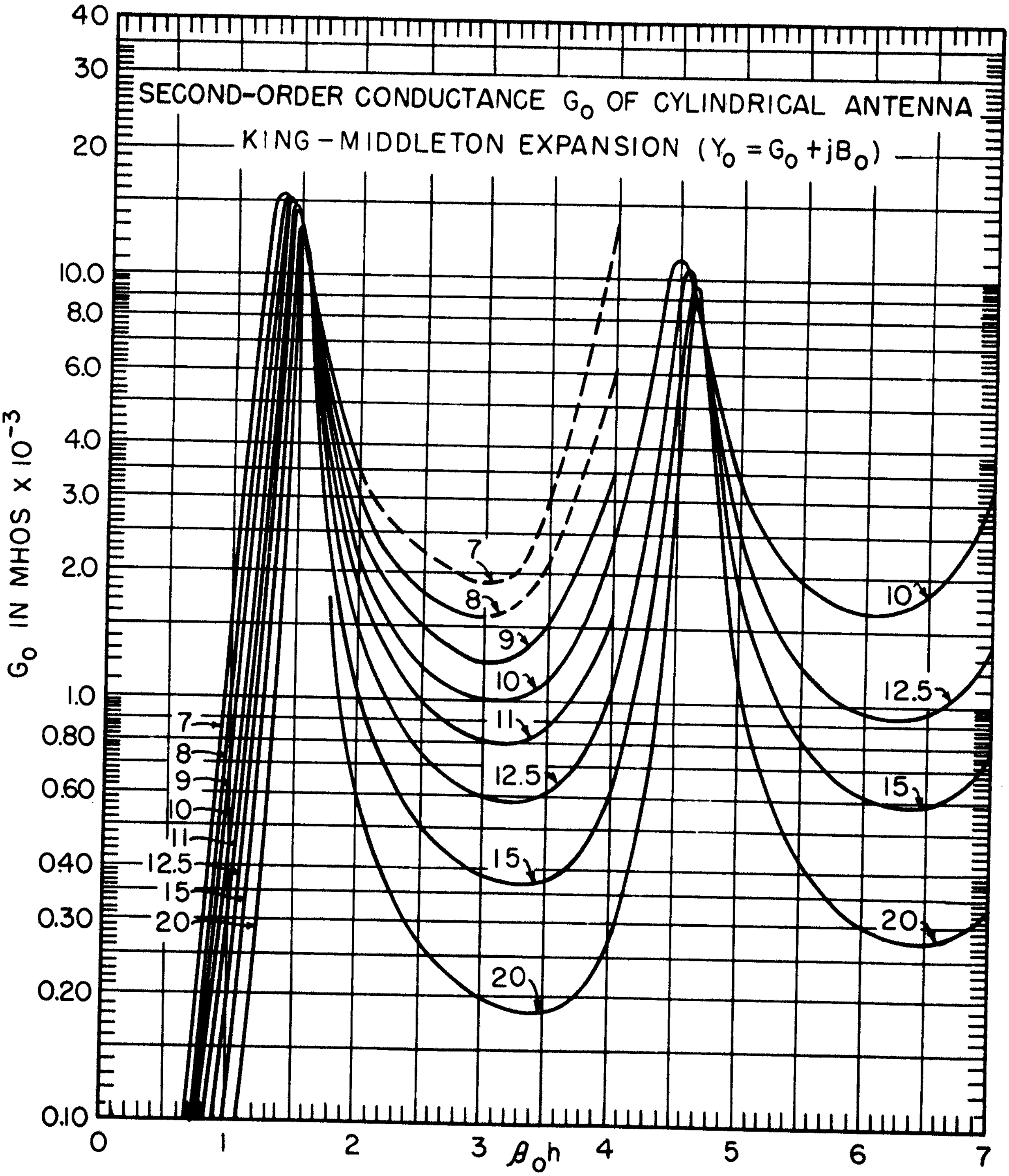
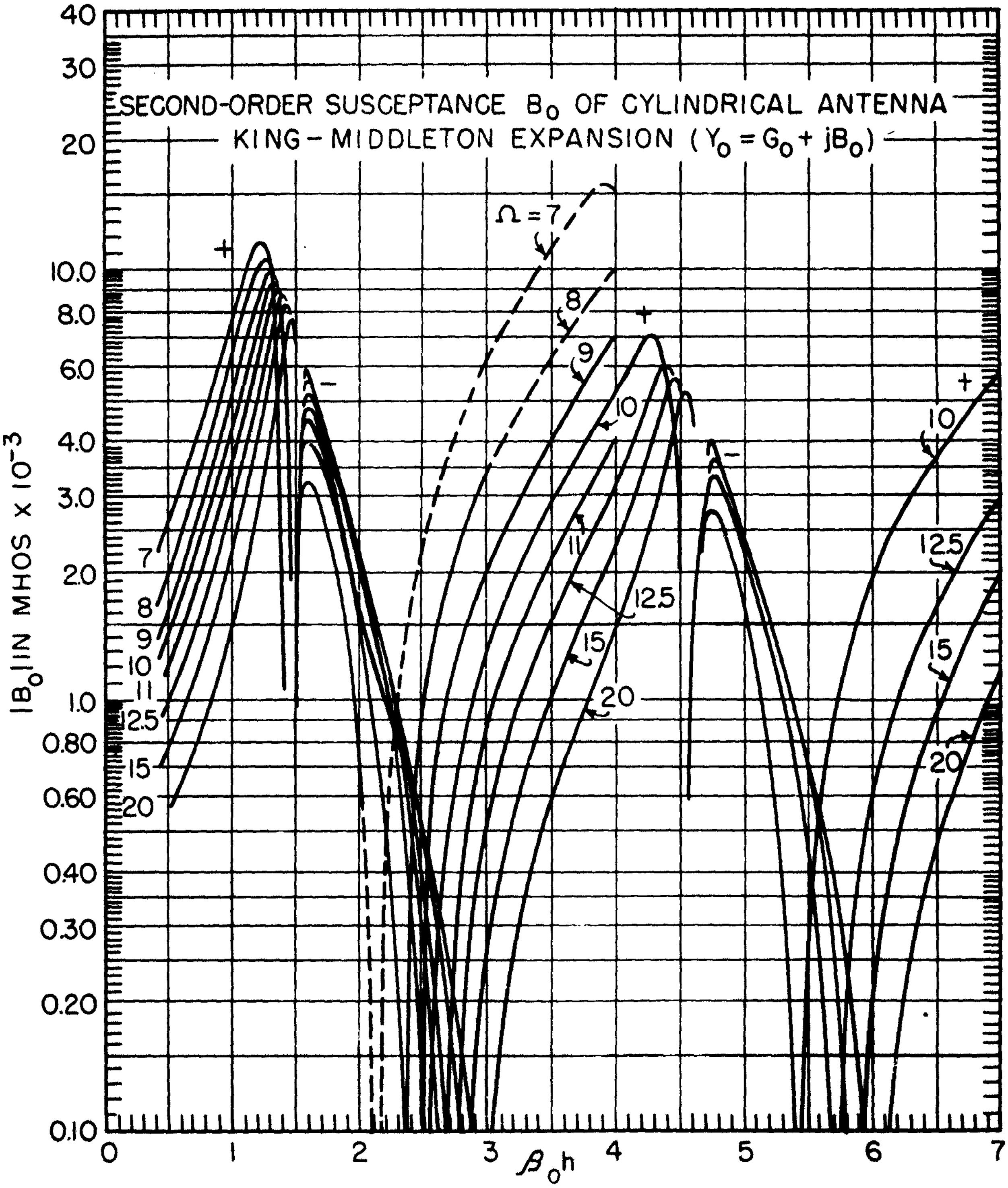

SECOND-ORDER CONDUCTANCE G_0 OF CYLINDRICAL ANTENNA
KING-MIDDLETON EXPANSION ($Y_0 = G_0 + jB_0$)
G_0 IN MHOS x 10^{-3}
$\beta_0 h$
40
30
20
10.0
8.0
6.0
4.0
3.0
2.0
1.0
0.80
0.60
0.40
0.30
0.20
0.10
0
1
2
3
4
5
6
7
7
8
9
10
11
12.5
15
20

FIGURE 11



SECOND-ORDER SUSCEPTANCE B_0 OF CYLINDRICAL ANTENNA
KING-MIDDLETON EXPANSION ($Y_0 = G_0 + jB_0$)
$|B_0|$ IN MHOS x 10^{-3}
$\beta_0 h$
$\Omega = 7$
8
9
10
11
12.5
15
20
+
−
40
30
20
10.0
8.0
6.0
4.0
3.0
2.0
1.0
0.80
0.60
0.40
0.30
0.20
0.10
0
1
2
3
4
5
6
7

FIGURE 12

KING-MIDDLETON SECOND-ORDER SUSCEPTANCE B_0 IN $Y_0 = G_0 + jB_0$

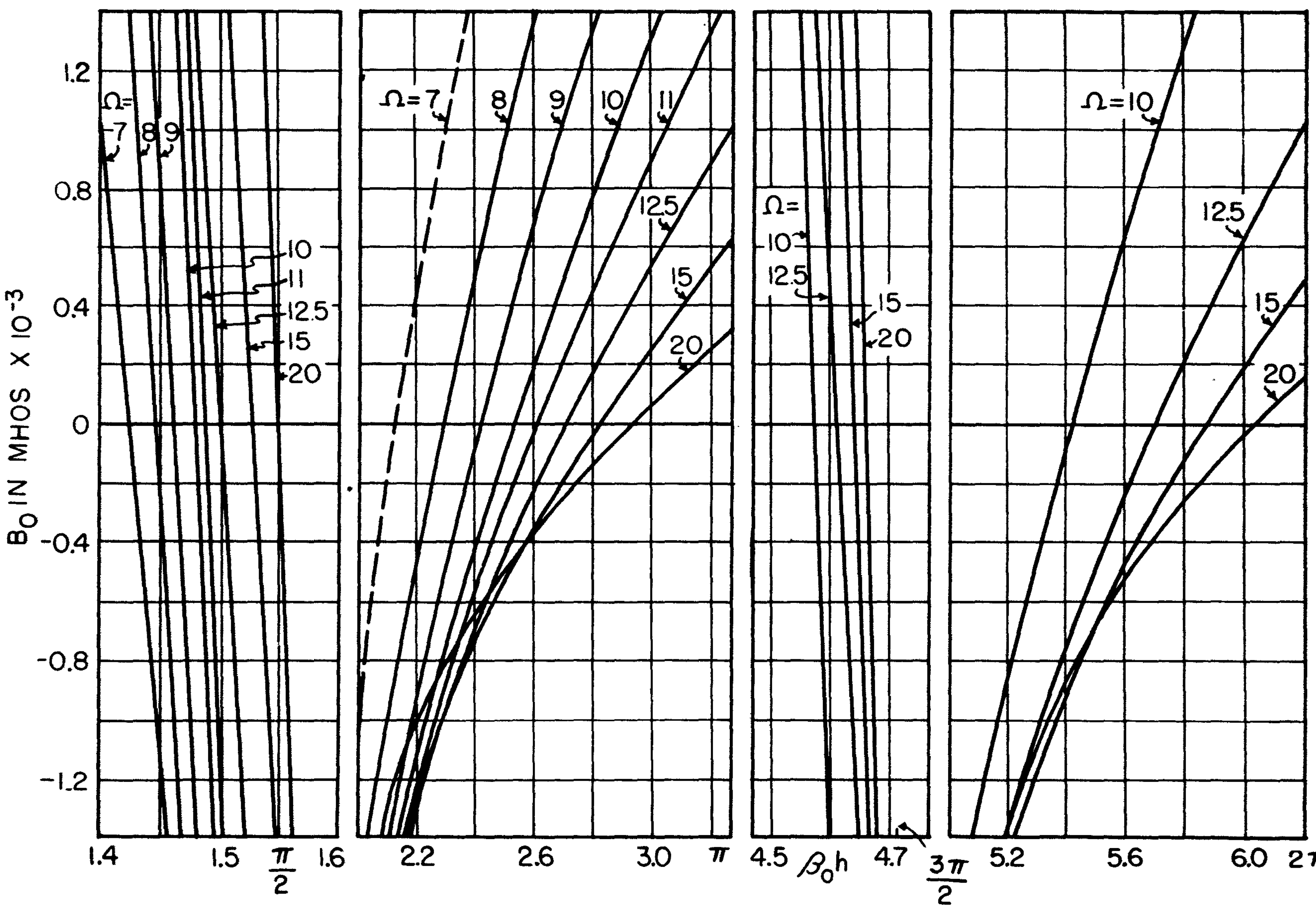
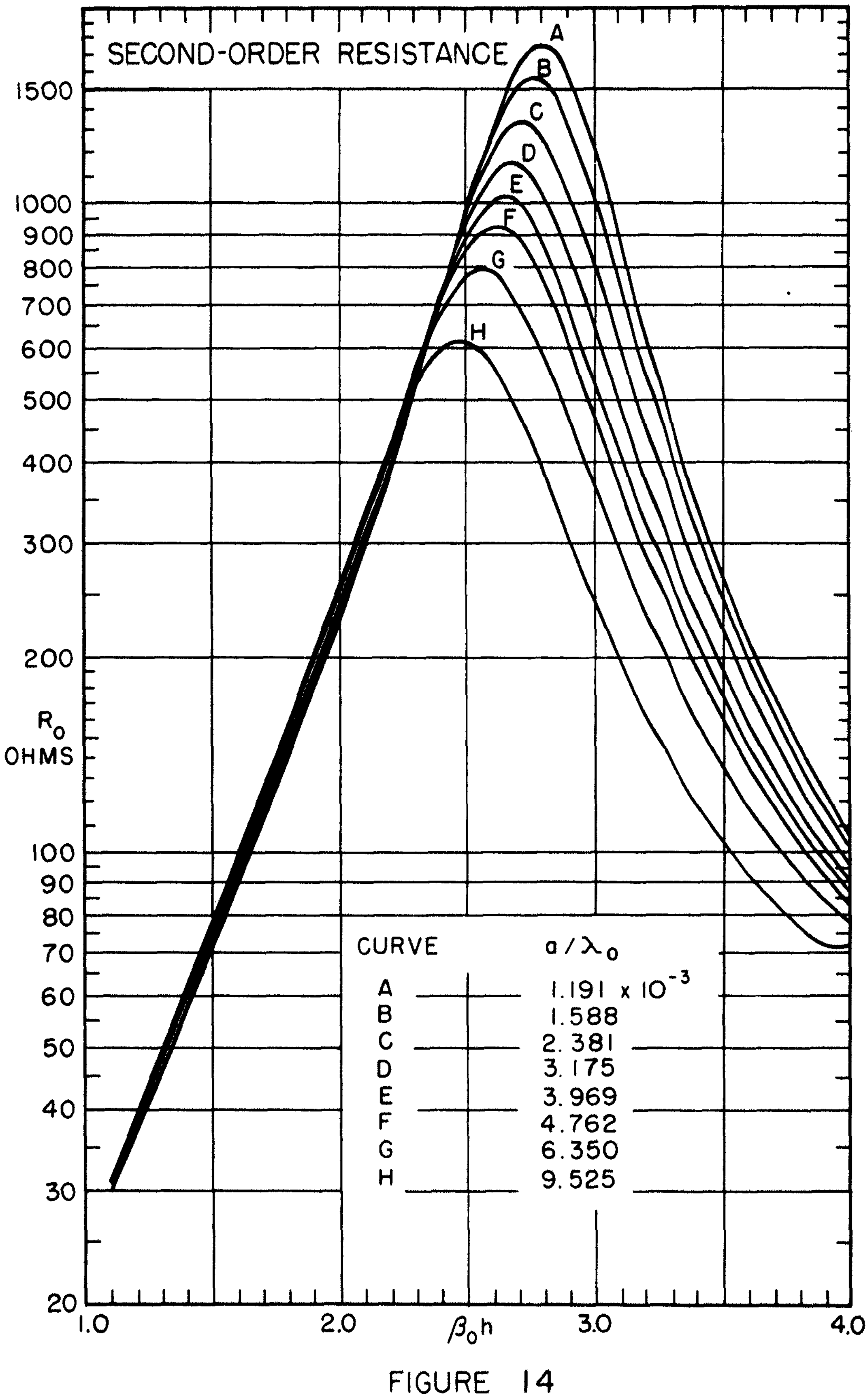
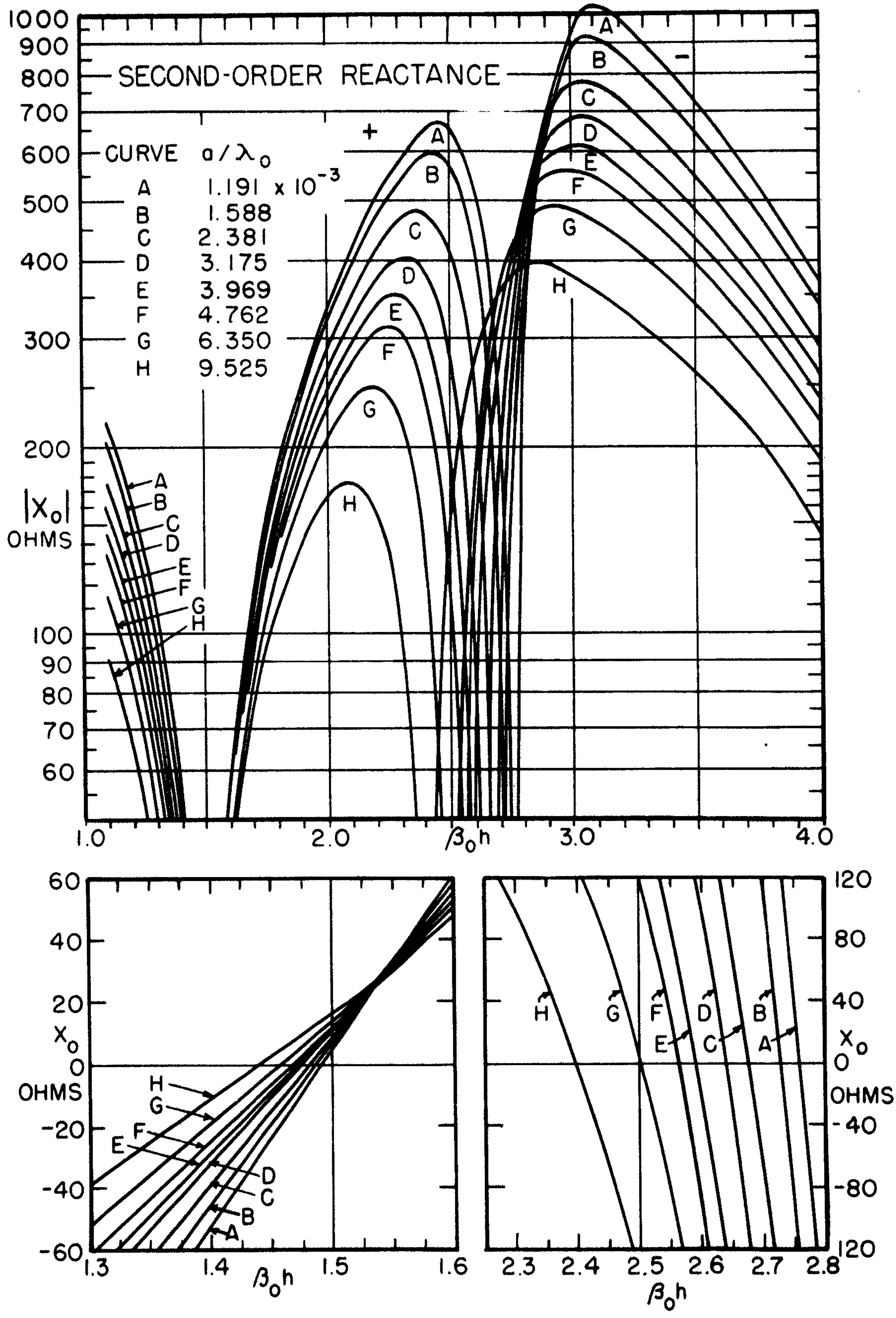


FIGURE 13



SECOND-ORDER RESISTANCE
A
B
C
D
E
F
G
H
1500
1000
900
800
700
600
500
400
300
200
R_0
OHMS
100
90
80
70
60
50
40
30
20
1.0
2.0
$\beta_0 h$
3.0
4.0
CURVE
a/λ_0
A 1.191 x 10^{-3}
B 1.588
C 2.381
D 3.175
E 3.969
F 4.762
G 6.350
H 9.525

FIGURE 14



SECOND-ORDER REACTANCE
CURVE a/λ_0
A 1.191 x 10^{-3}
B 1.588
C 2.381
D 3.175
E 3.969
F 4.762
G 6.350
H 9.525
+
−
$|X_0|$ OHMS
$\beta_0 h$
X_0 OHMS
$\beta_0 h$
X_0 OHMS
$\beta_0 h$

FIGURE 15

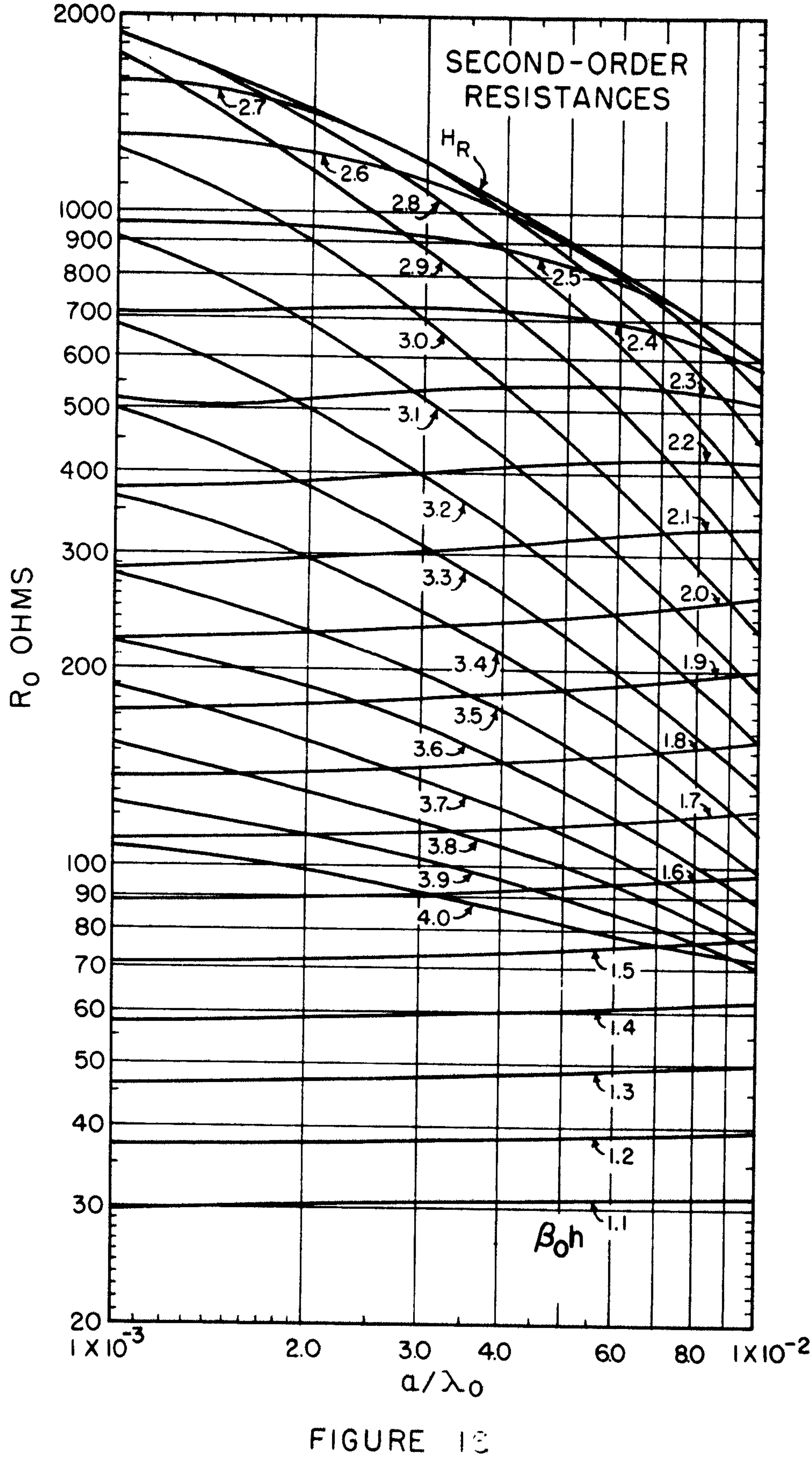
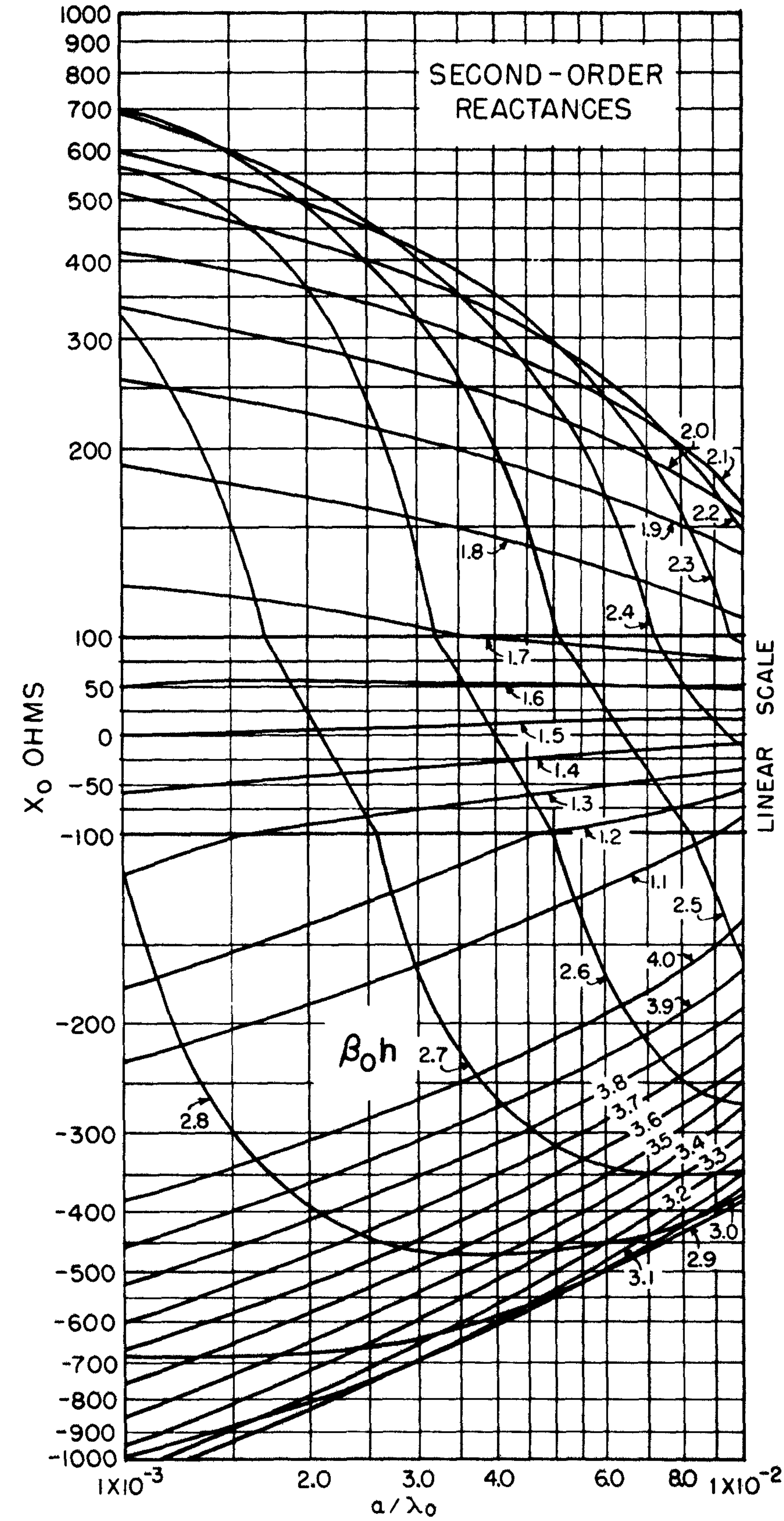

SECOND-ORDER RESISTANCES
H_R
2.7
2.6
2.8
2.9
2.5
3.0
2.4
2.3
3.1
2.2
3.2
2.1
3.3
2.0
3.4
1.9
3.5
3.6
1.8
3.7
1.7
3.8
1.6
3.9
4.0
1.5
1.4
1.3
1.2
1.1
$\beta_0 h$
2000
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
R_0 OHMS
1×10^{-3}
2.0
3.0
4.0
6.0
8.0
1×10^{-2}
a/λ_0

FIGURE 13



SECOND-ORDER REACTANCES
X_0 OHMS
LINEAR SCALE
$\beta_0 h$
a/λ_0
1000
900
800
700
600
500
400
300
200
100
50
0
-50
-100
-200
-300
-400
-500
-600
-700
-800
-900
-1000
1X10^{-3}
2.0
3.0
4.0
6.0
8.0
1X10^{-2}
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3.0
3.1
3.2
3.3
3.4
3.5
3.6
3.7
3.8
3.9
4.0

FIGURE 17

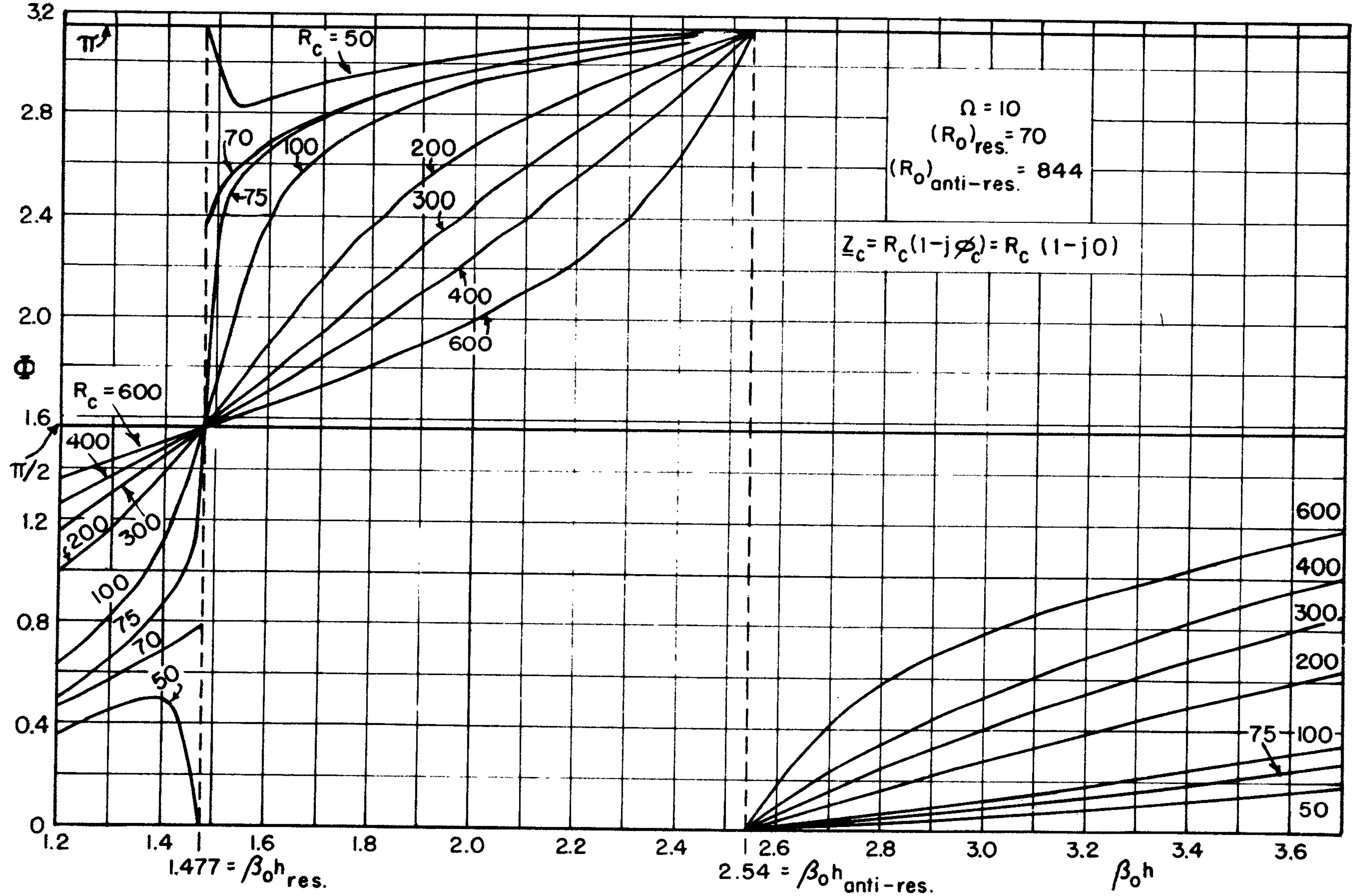


FIG. 18a TERMINAL FUNCTION, Φ

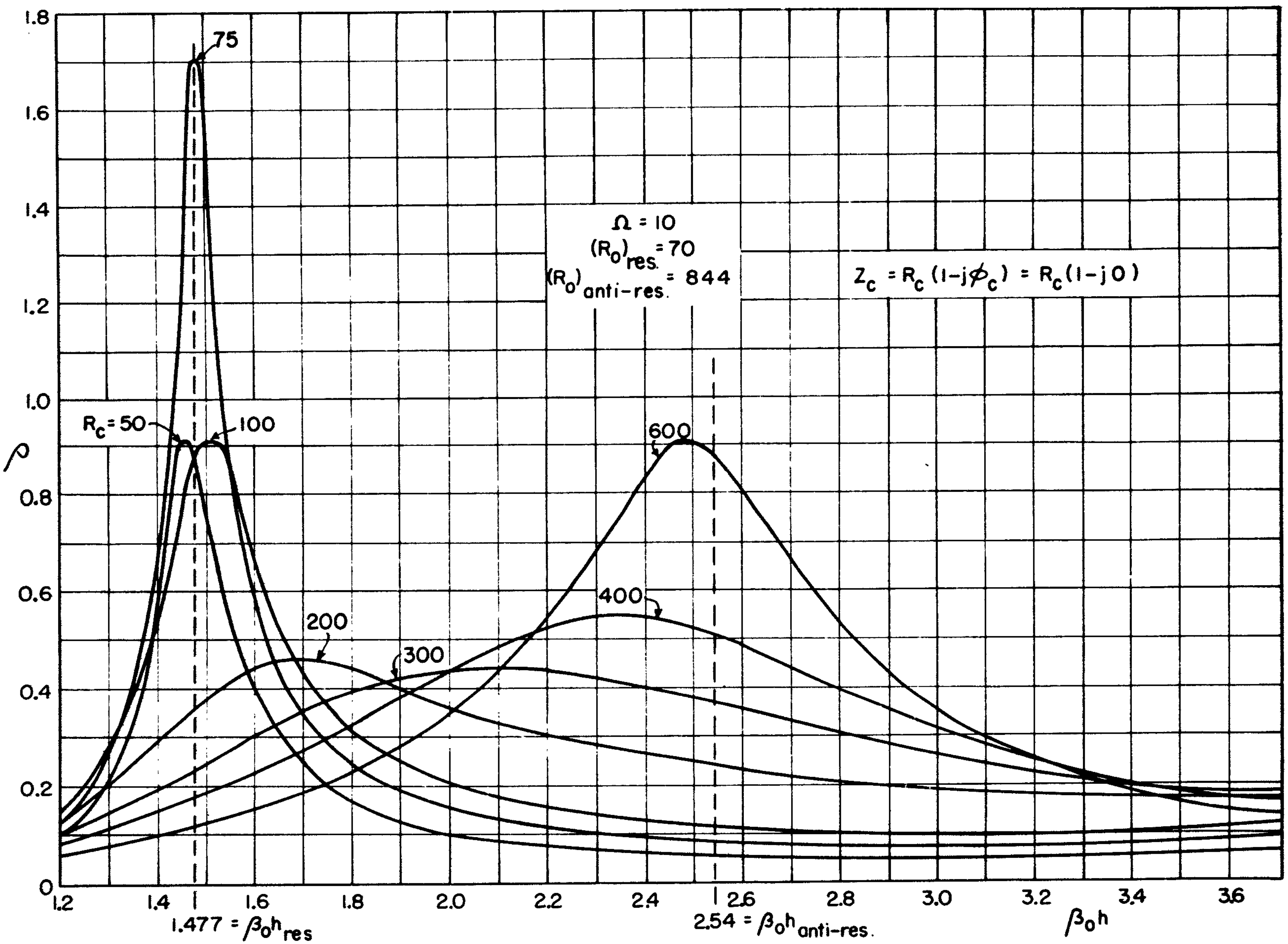


FIG. 18b TERMINAL FUNCTION, ρ

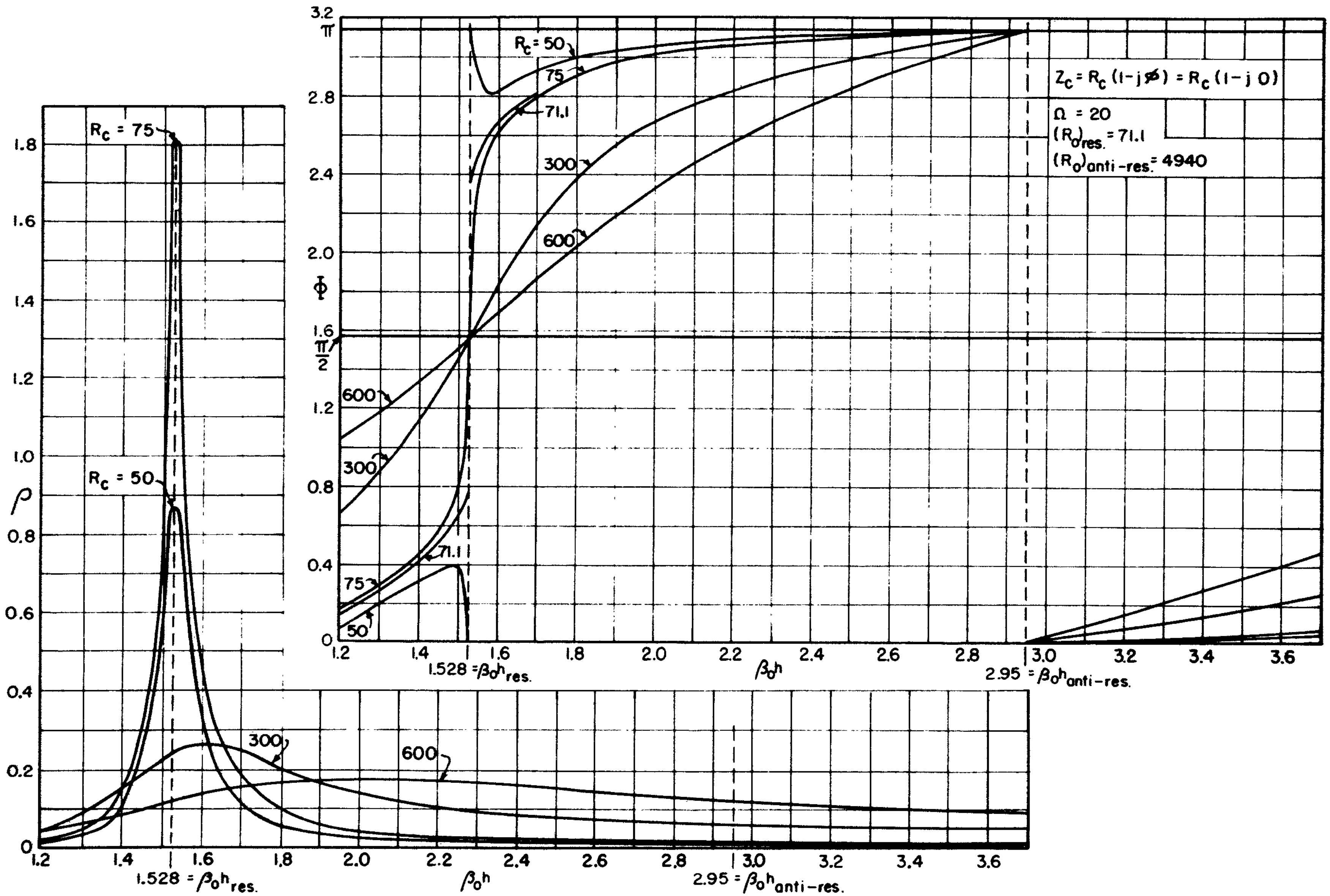
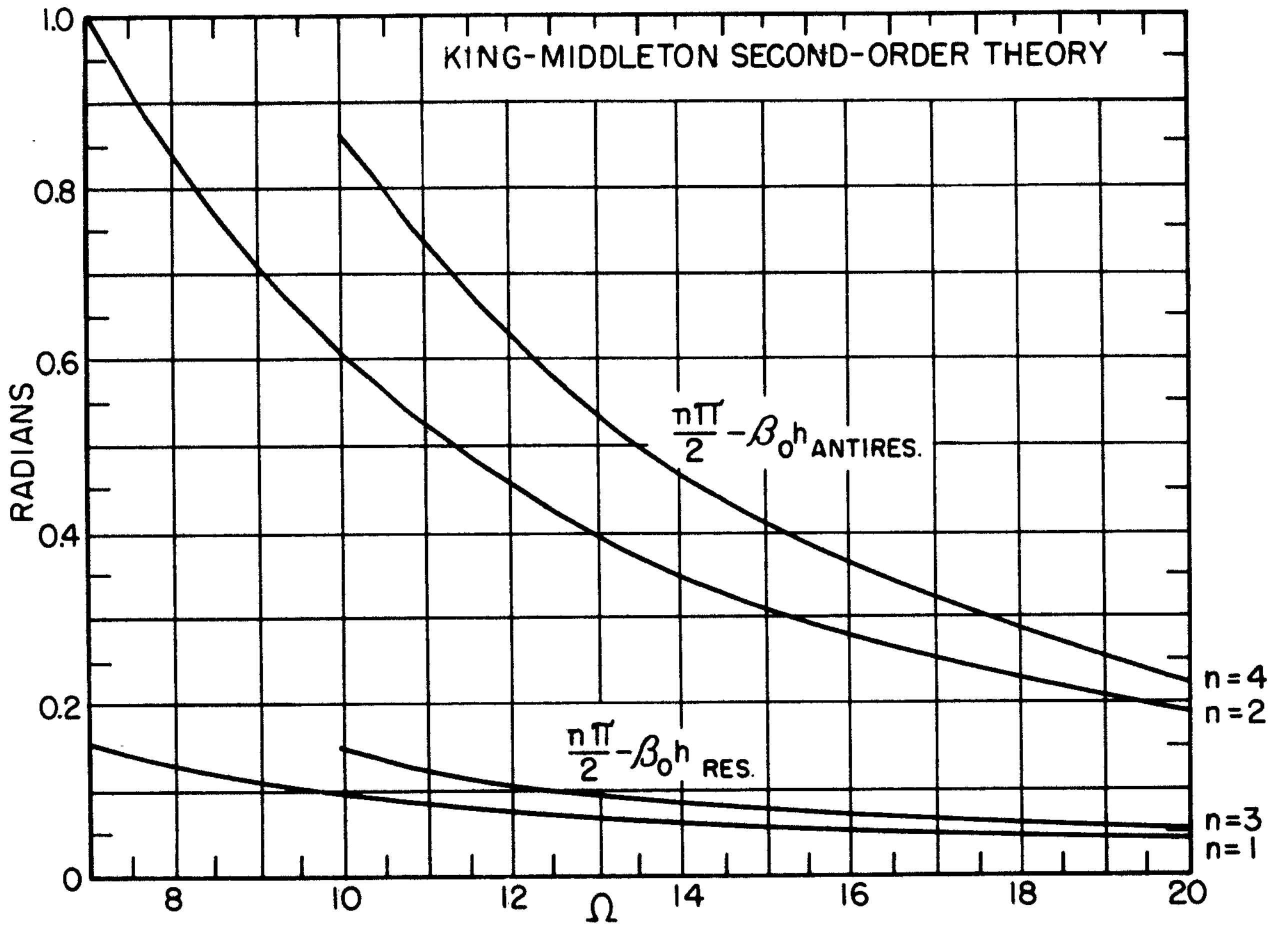


FIG. 19 TERMINAL FUNCTIONS, Φ AND ρ



KING-MIDDLETON SECOND-ORDER THEORY
RADIANS
1.0
0.8
0.6
0.4
0.2
0
8
10
12
14
16
18
20
Ω
$\frac{n\pi}{2} - \beta_0 h_{\text{ANTIRES.}}$
$\frac{n\pi}{2} - \beta_0 h_{\text{RES.}}$
n = 4
n = 2
n = 3
n = 1

FIGURE 20

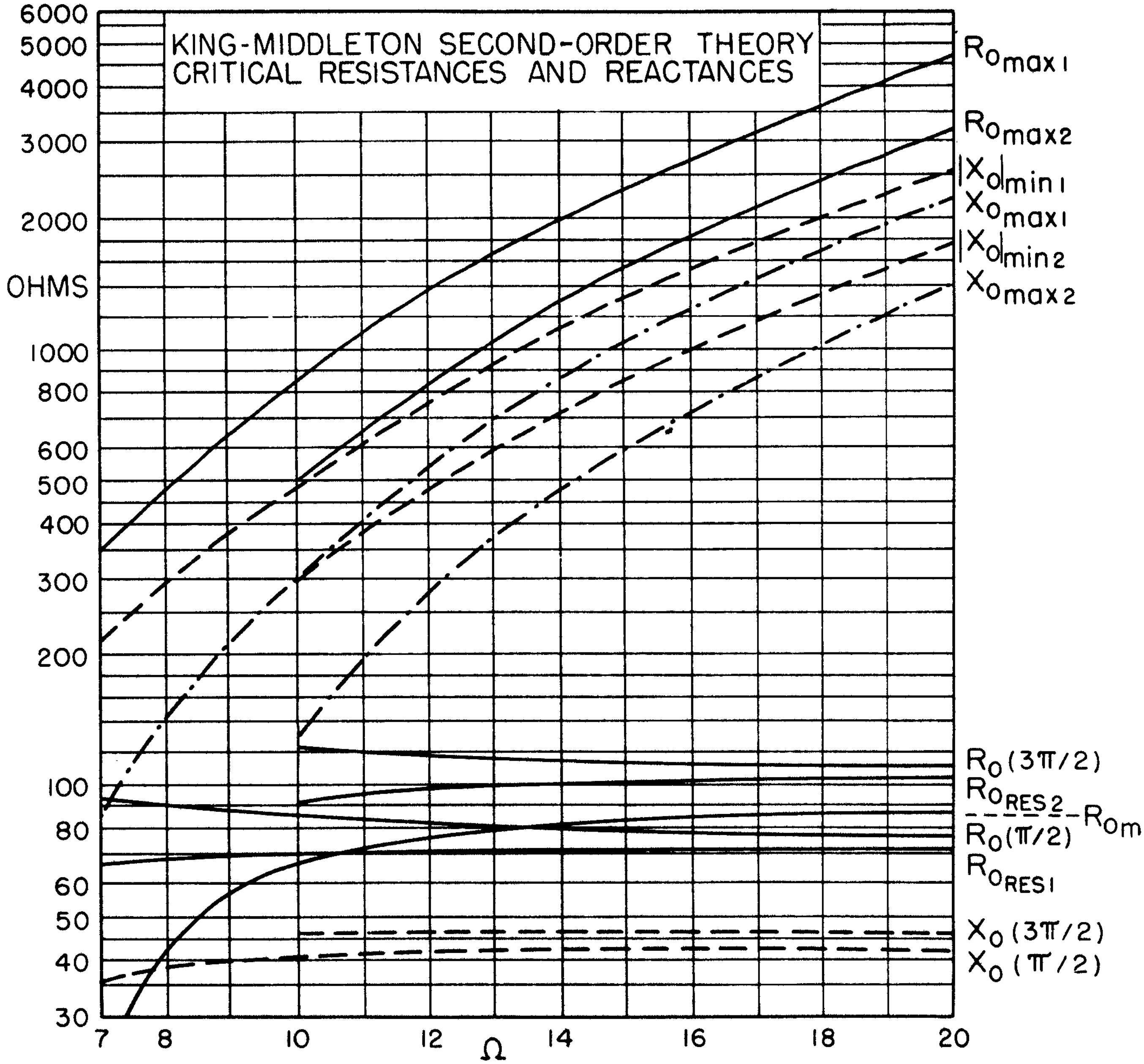

KING-MIDDLETON SECOND-ORDER THEORY
CRITICAL RESISTANCES AND REACTANCES
OHMS
6000
5000
4000
3000
2000
1000
800
600
500
400
300
200
100
80
60
50
40
30
7
8
10
12
Ω
14
16
18
20
$R_{O\,max1}$
$R_{O\,max2}$
$|X_O|_{min1}$
$X_{O\,max1}$
$|X_O|_{min2}$
$X_{O\,max2}$
$R_O(3\pi/2)$
$R_{O\,RES2}$
R_{Om}
$R_O(\pi/2)$
$R_{O\,RES1}$
$X_O(3\pi/2)$
$X_O(\pi/2)$

FIGURE 21

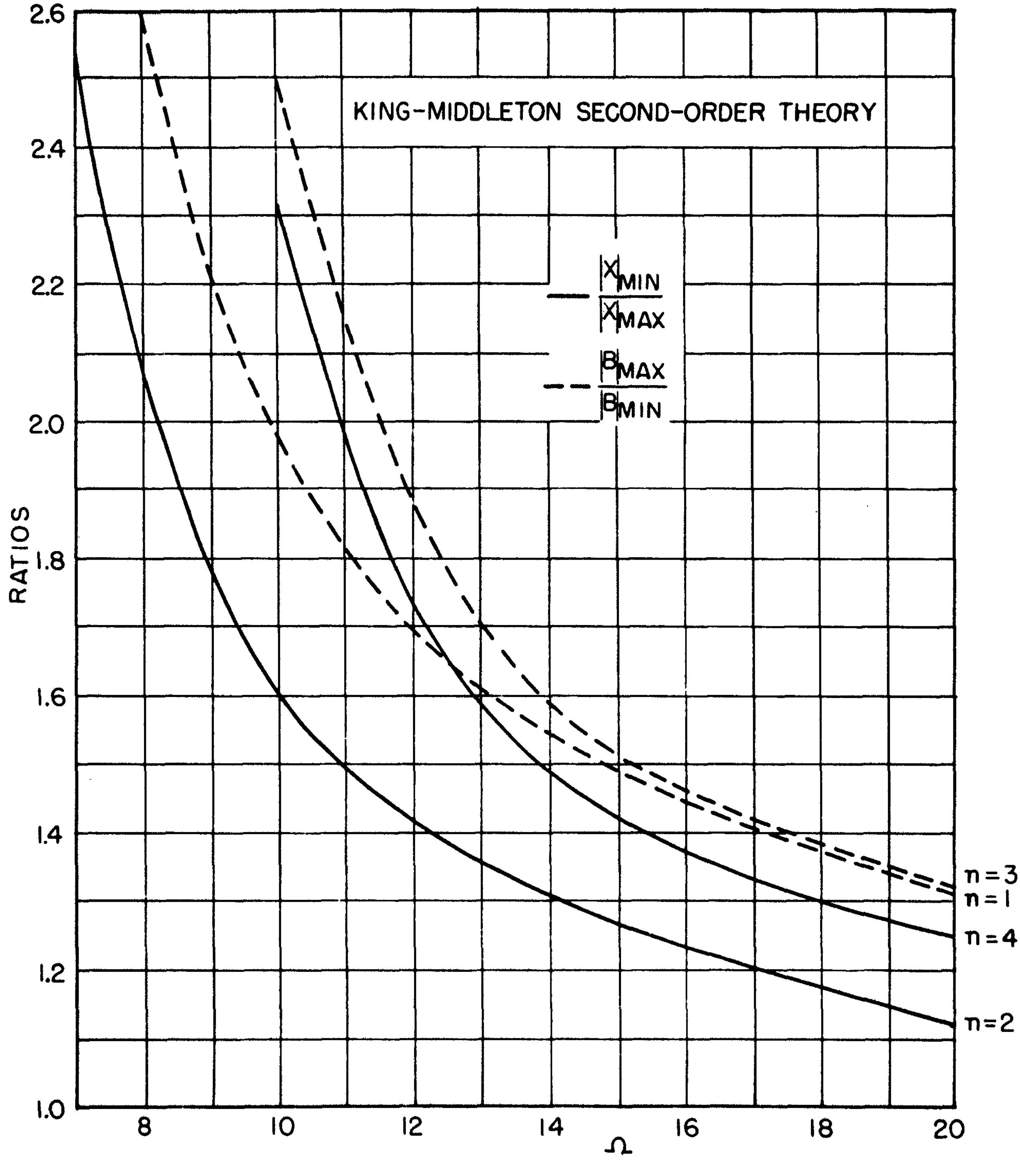

KING-MIDDLETON SECOND-ORDER THEORY
|X|MIN / |X|MAX
|B|MAX / |B|MIN
RATIOS
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
8
10
12
14
16
18
20
Ω
n=3
n=1
n=4
n=2

FIGURE 22

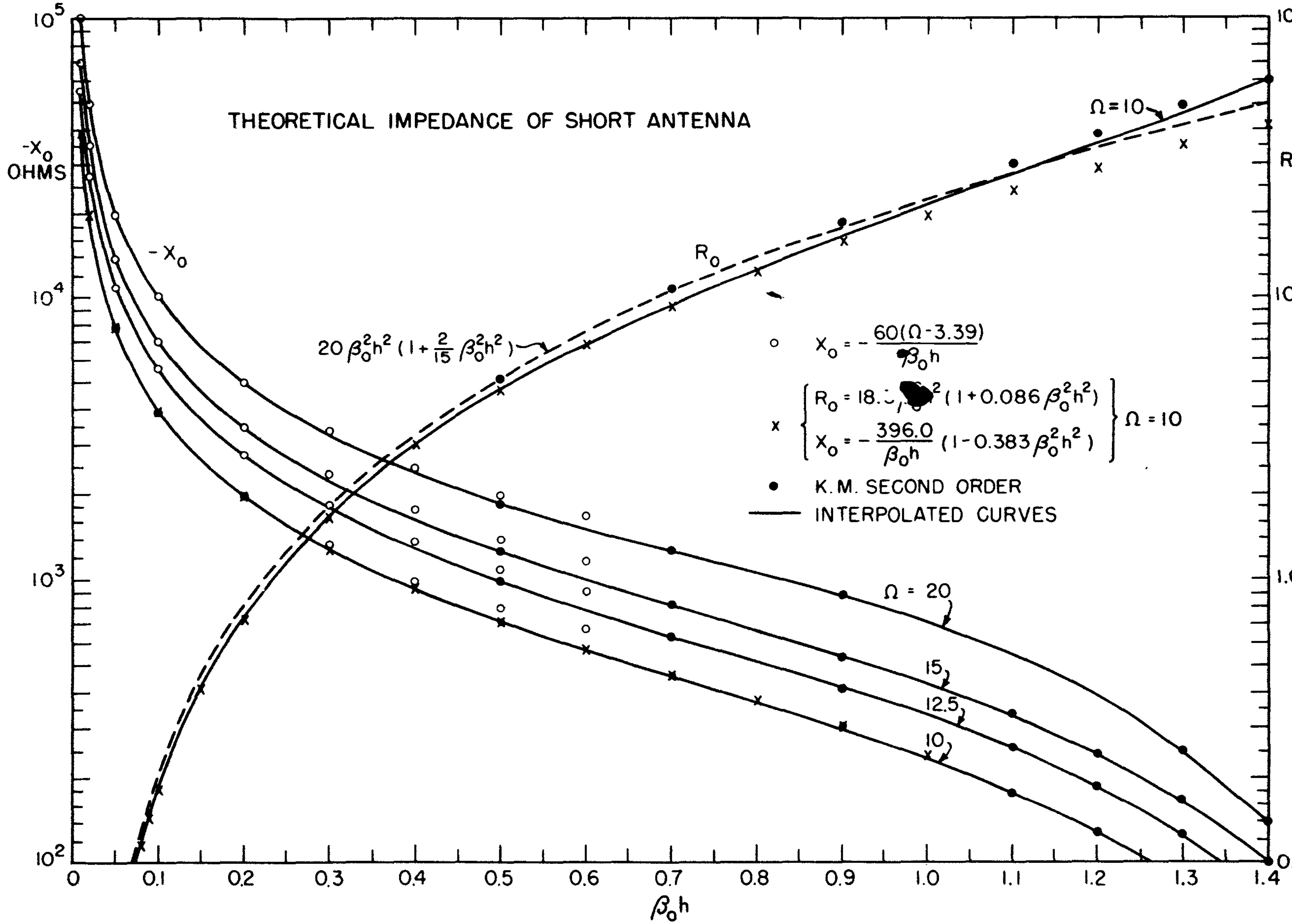

THEORETICAL IMPEDANCE OF SHORT ANTENNA
-X o OHMS
R o
- X o
R o
Ω = 10
20 β o 2 h 2 (1 + 2/15 β o 2 h 2)
X o = - 60(Ω-3.39)/β o h
R o = 18.
(1+0.086 β o 2 h 2)
X o = - 396.0/β o h (1-0.383 β o 2 h 2)
Ω = 10
K.M. SECOND ORDER
INTERPOLATED CURVES
Ω = 20
15
12.5
10
β o h

FIGURE 23

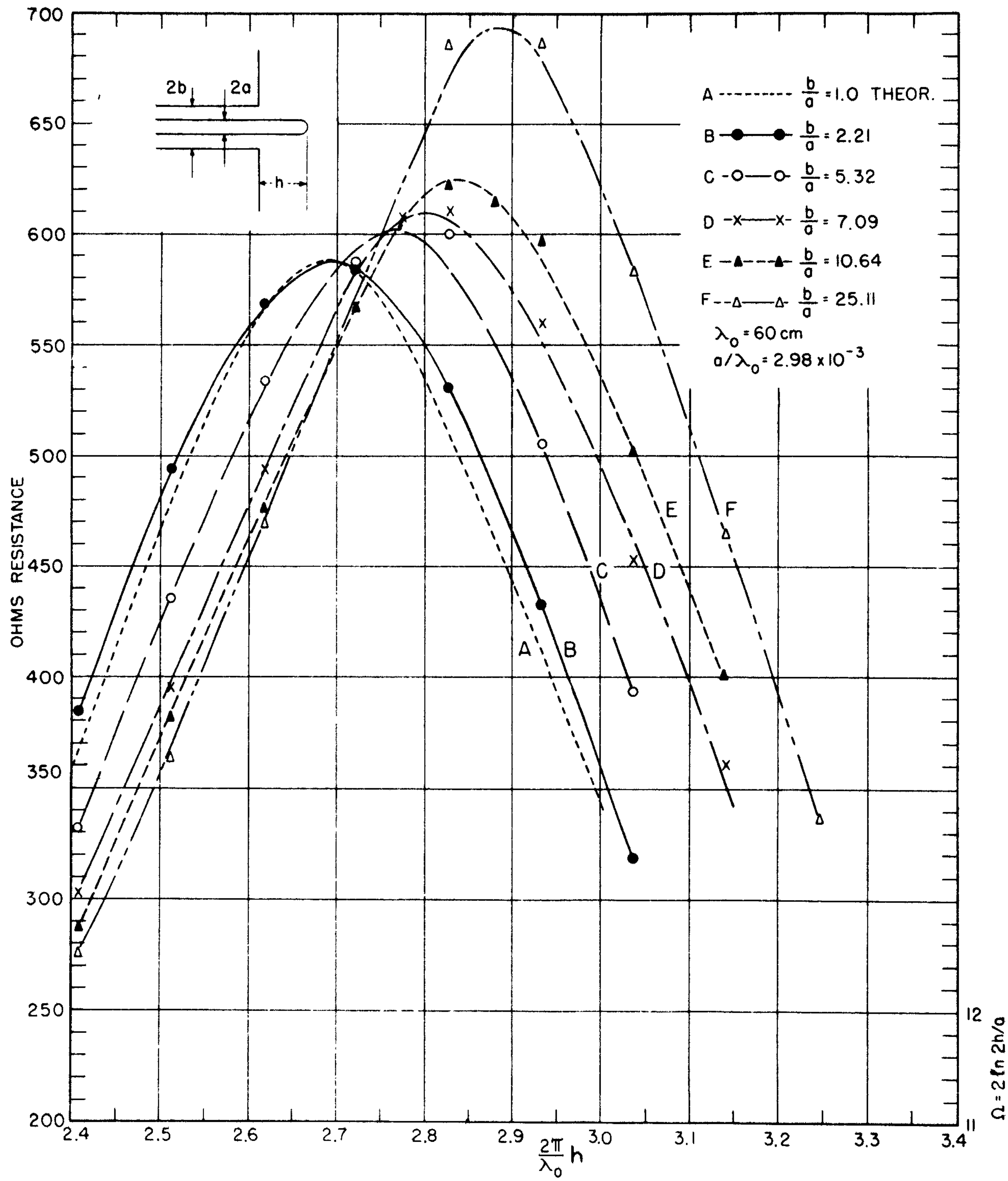


FIG. **24** RESISTANCE NEAR ANTIRESONANCE FOR A THIN ANTENNA

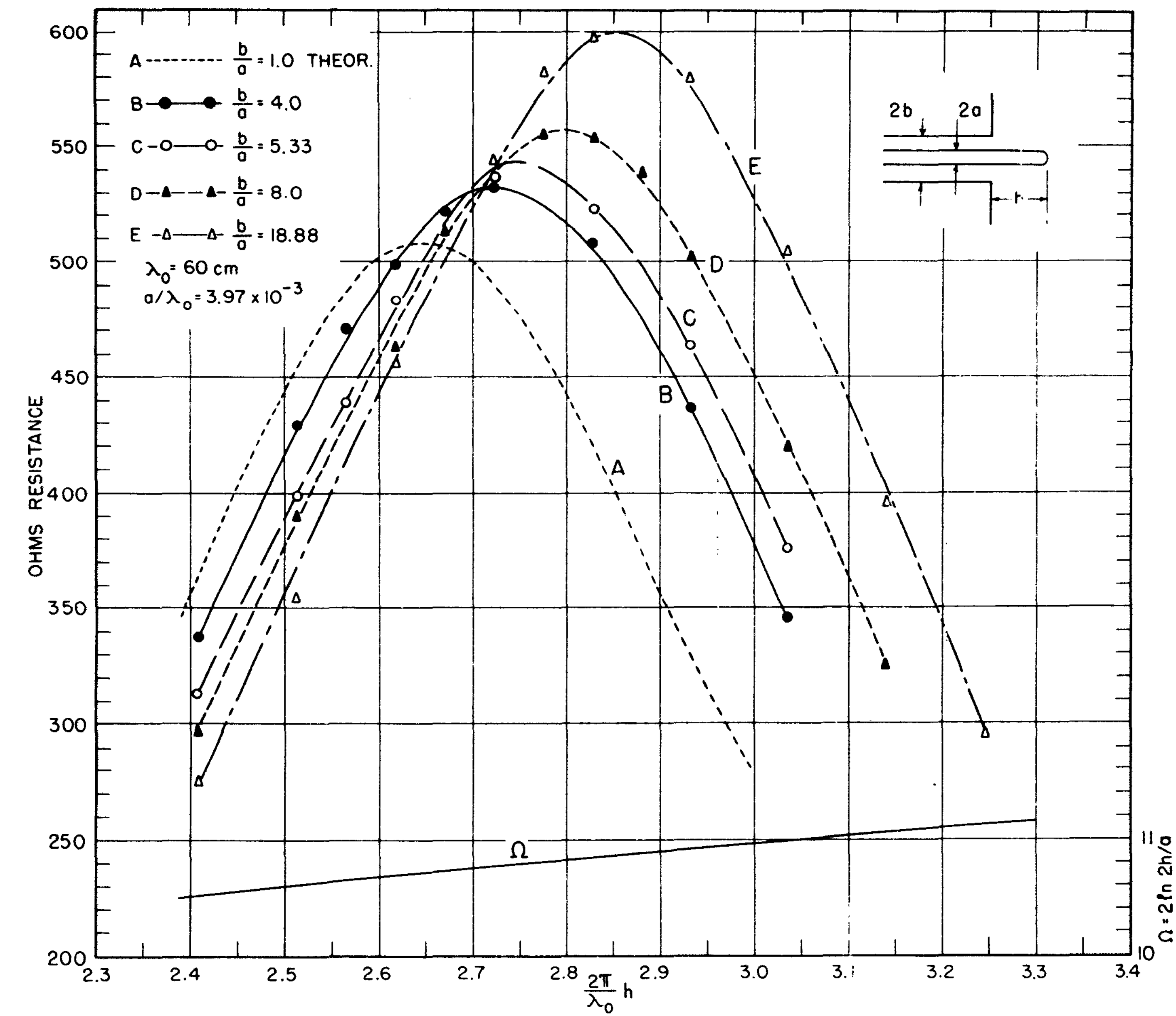


FIG. 25 RESISTANCE NEAR ANTIRESONANCE FOR A MODERATELY THIN ANTENNA

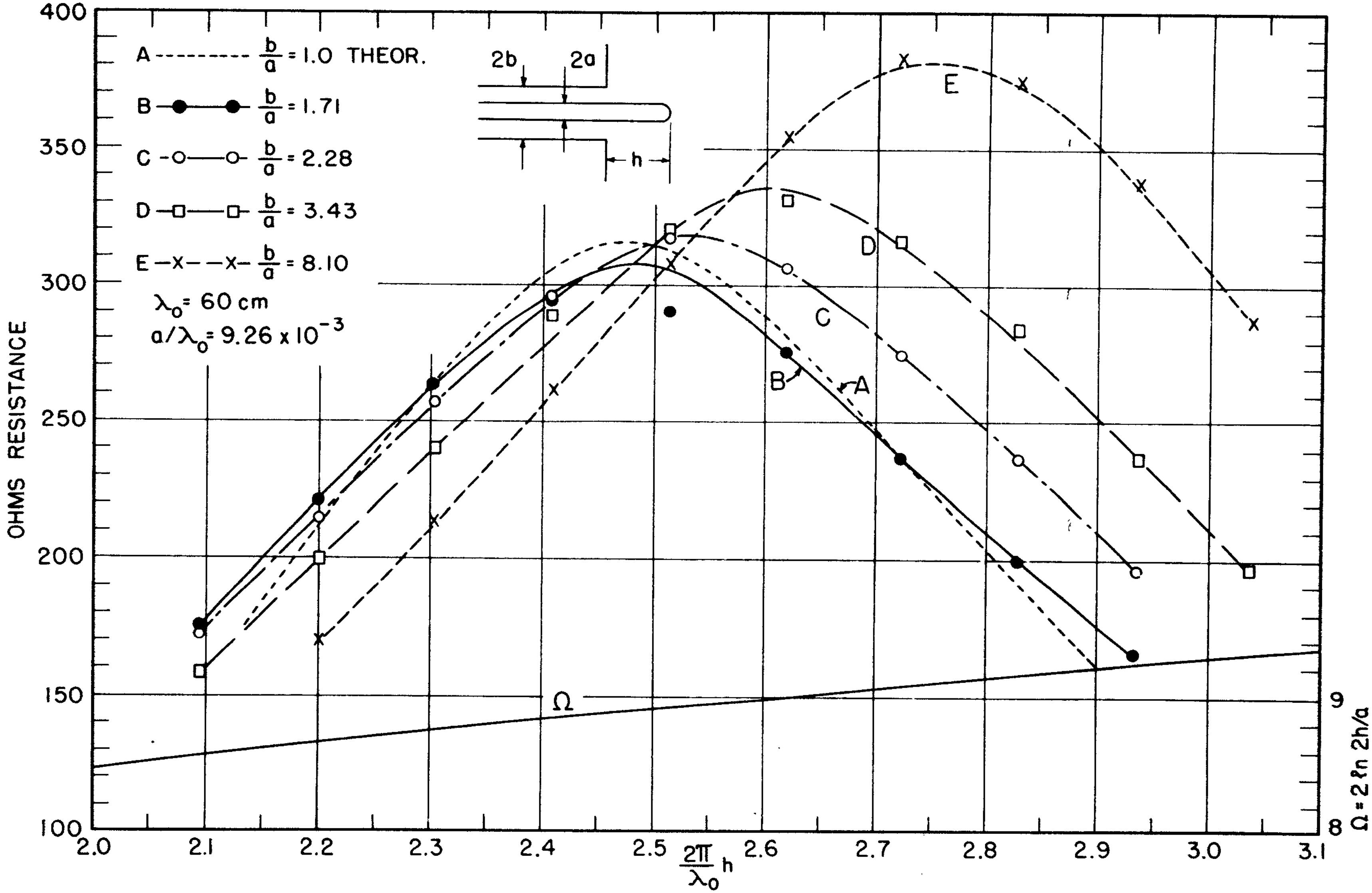


FIG. 26 RESISTANCE NEAR ANTIRESONANCE FOR A MODERATELY THICK ANTENNA

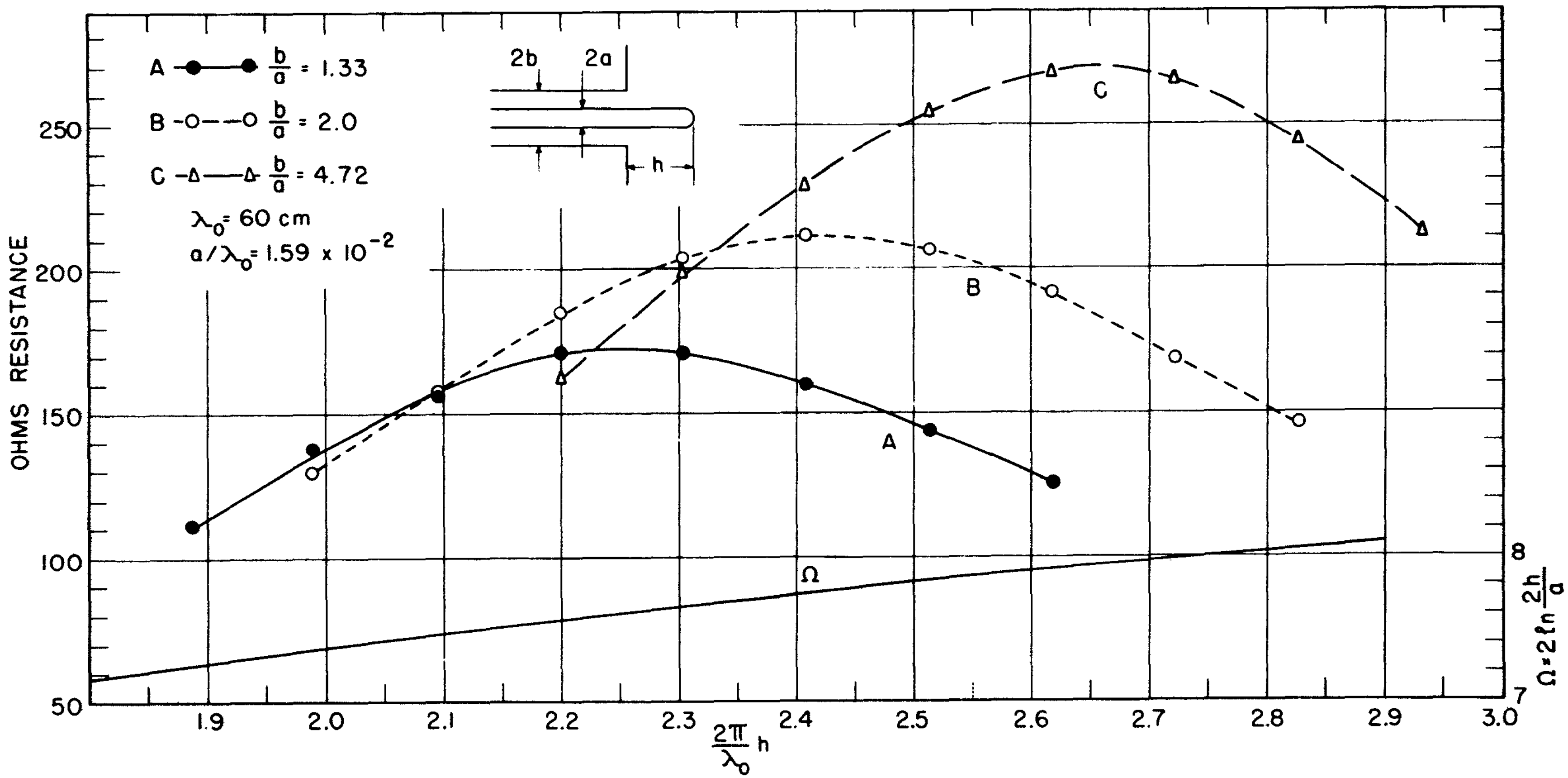


FIG. 27 MEASURED RESISTANCE NEAR ANTIRESONANCE FOR A THICK ANTENNA

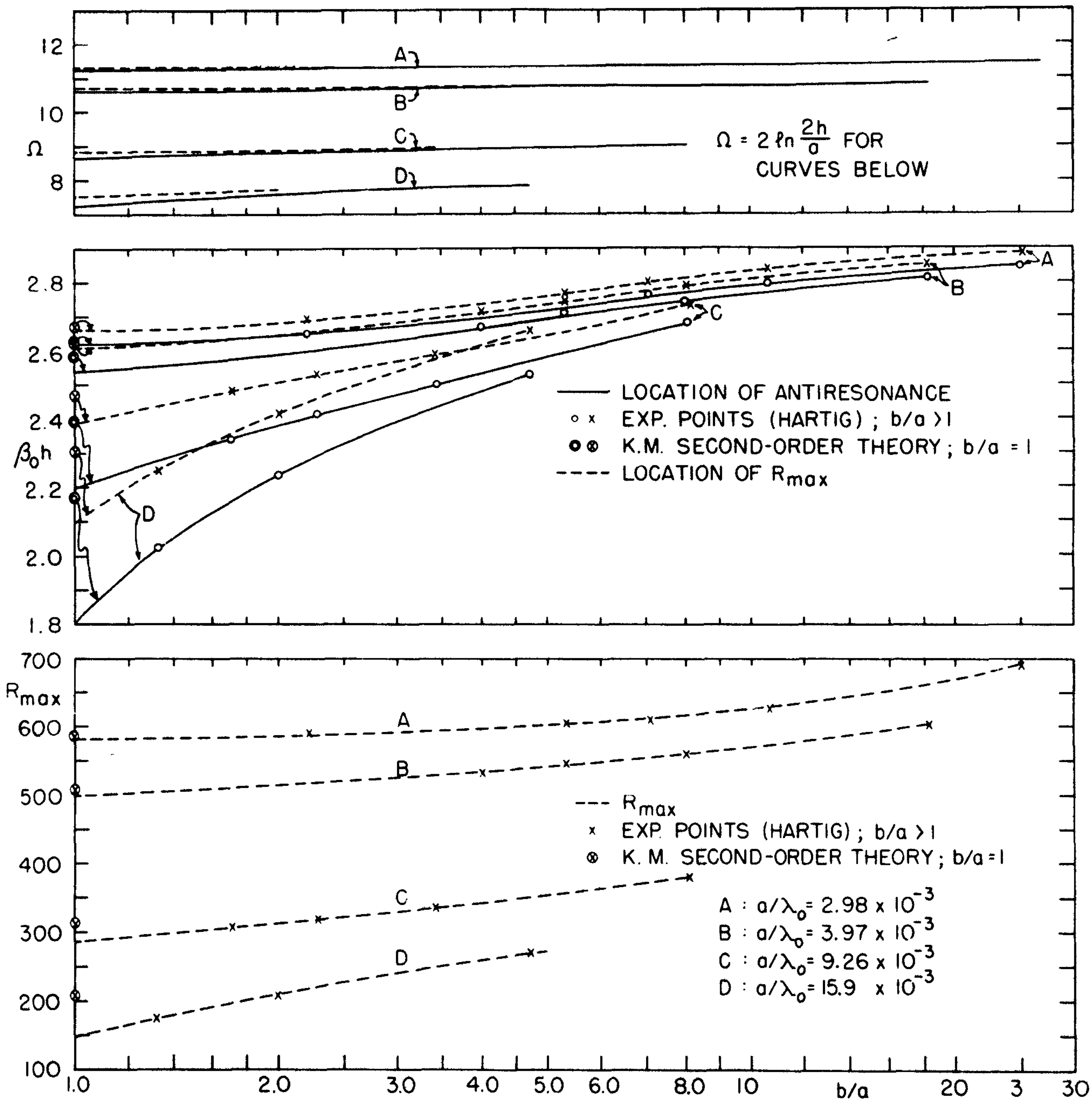


FIG. **28** SUMMARY OF CRITICAL VALUES NEAR ANTIRESONANCE

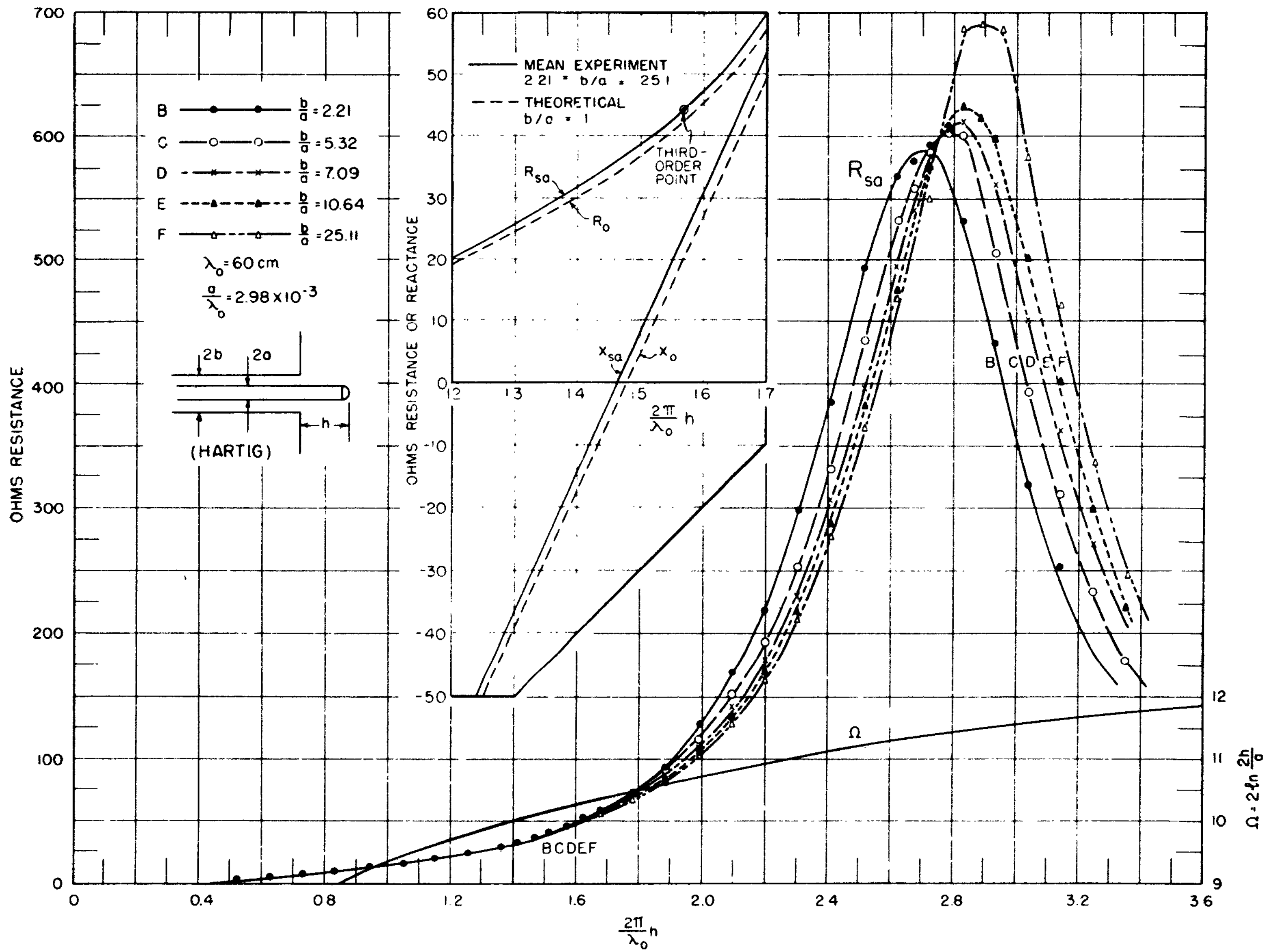


FIG. 29 MEASURED RESISTANCE OF A THIN ANTENNA

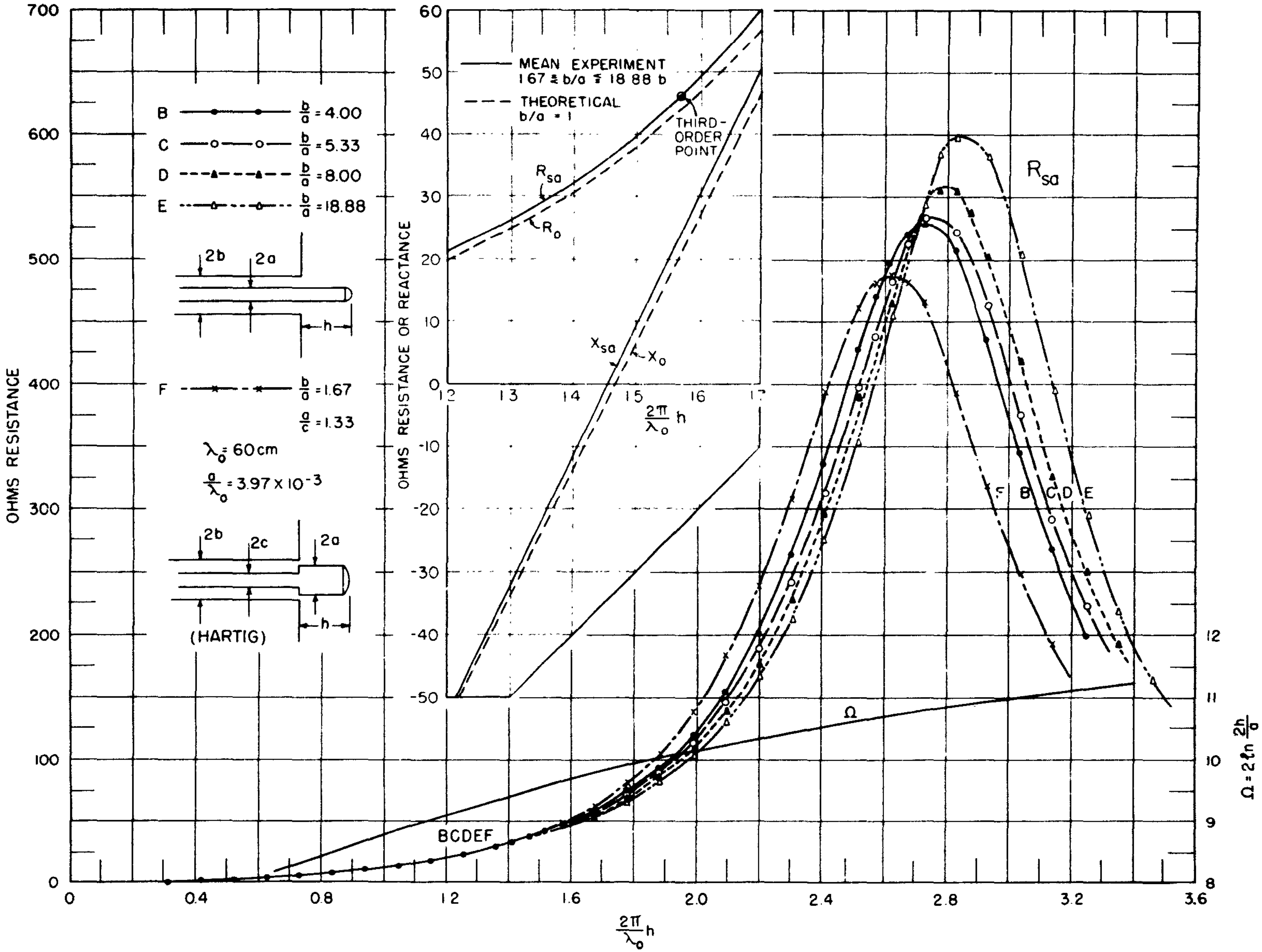


FIG. 30 MEASURED RESISTANCE OF A MODERATELY THIN ANTENNA

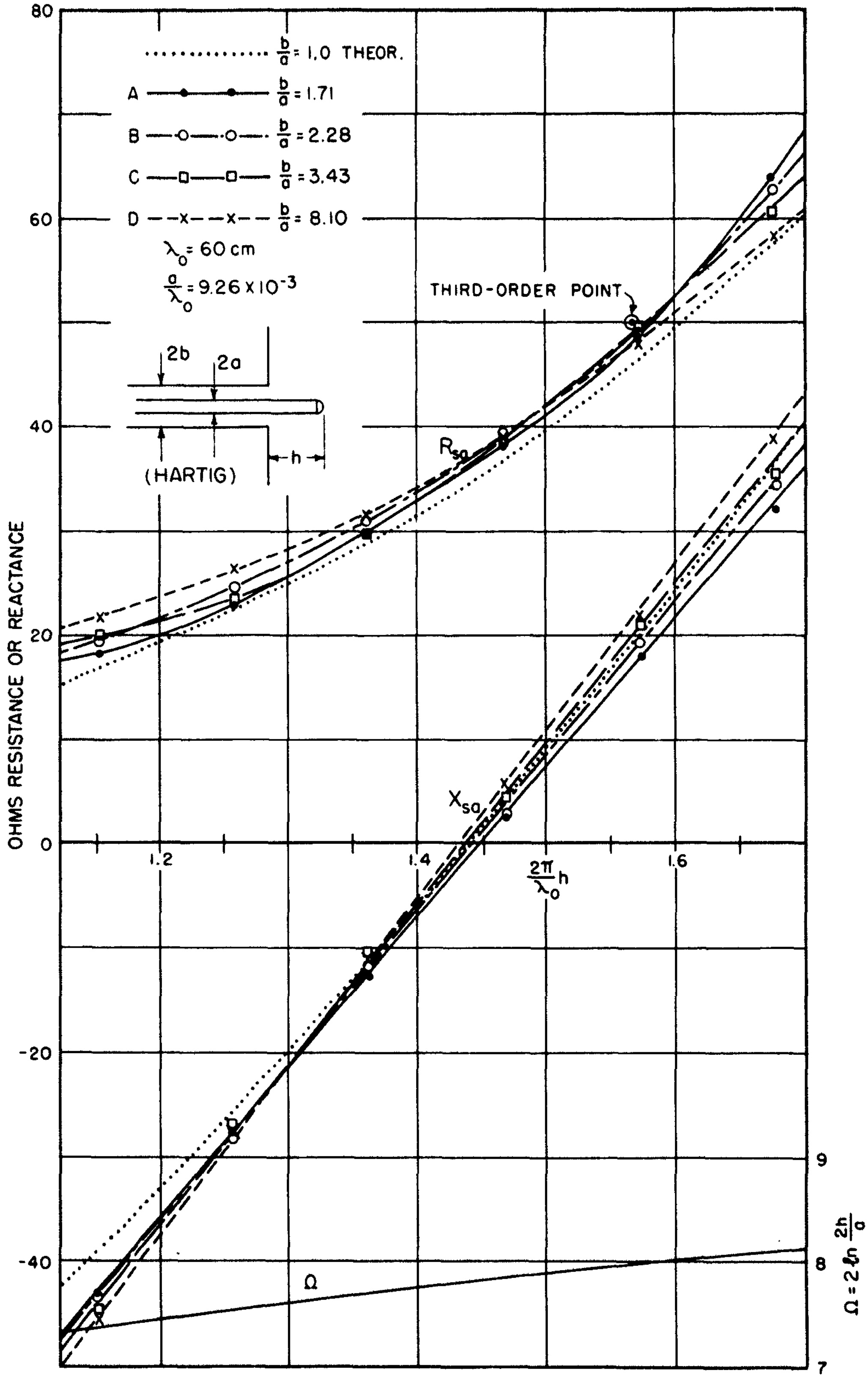


FIG. 31 IMPEDANCE NEAR RESONANCE FOR A MODERATELY THICK ANTENNA

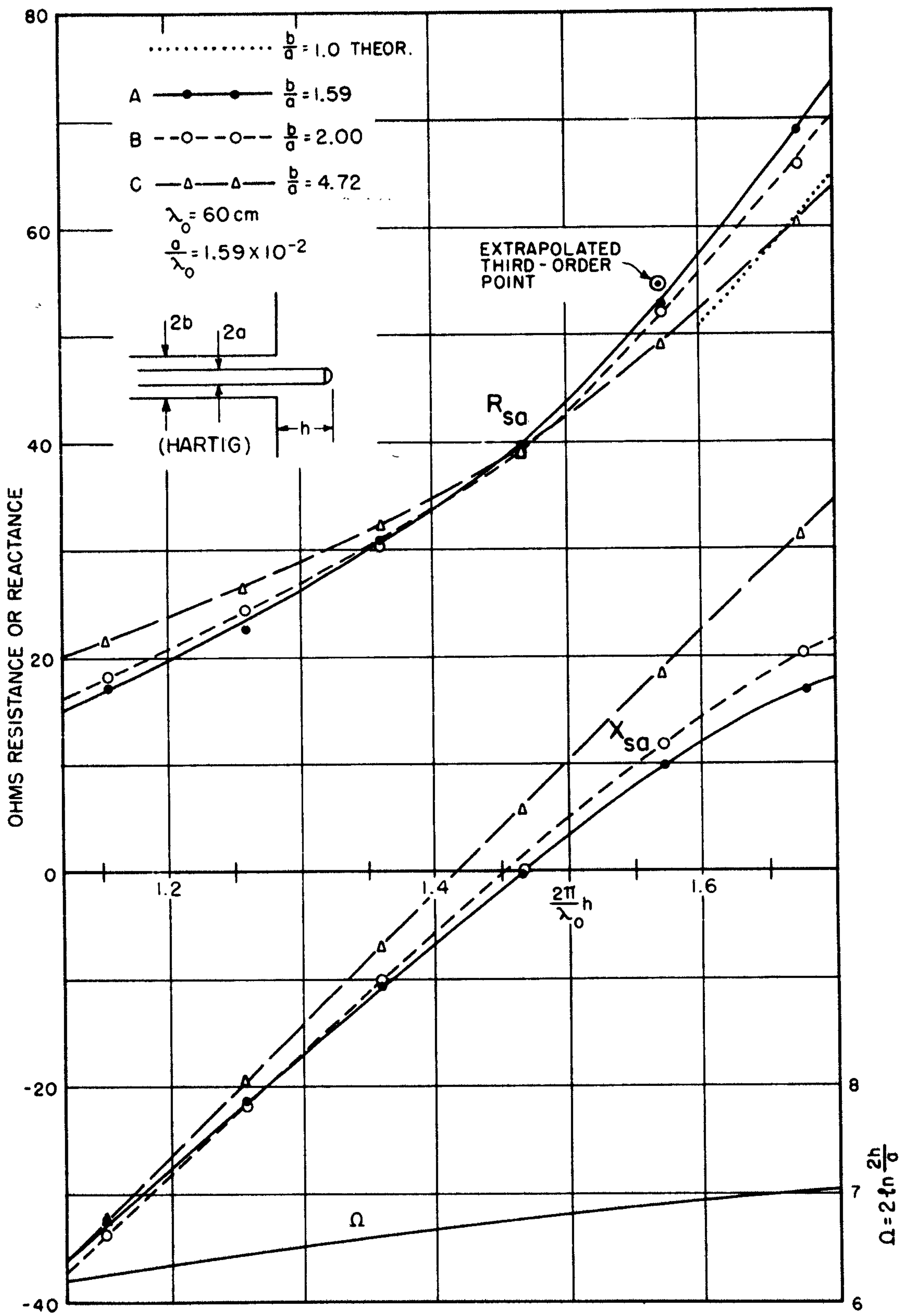


FIG. 32 IMPEDANCE NEAR RESONANCE FOR A THICK ANTENNA

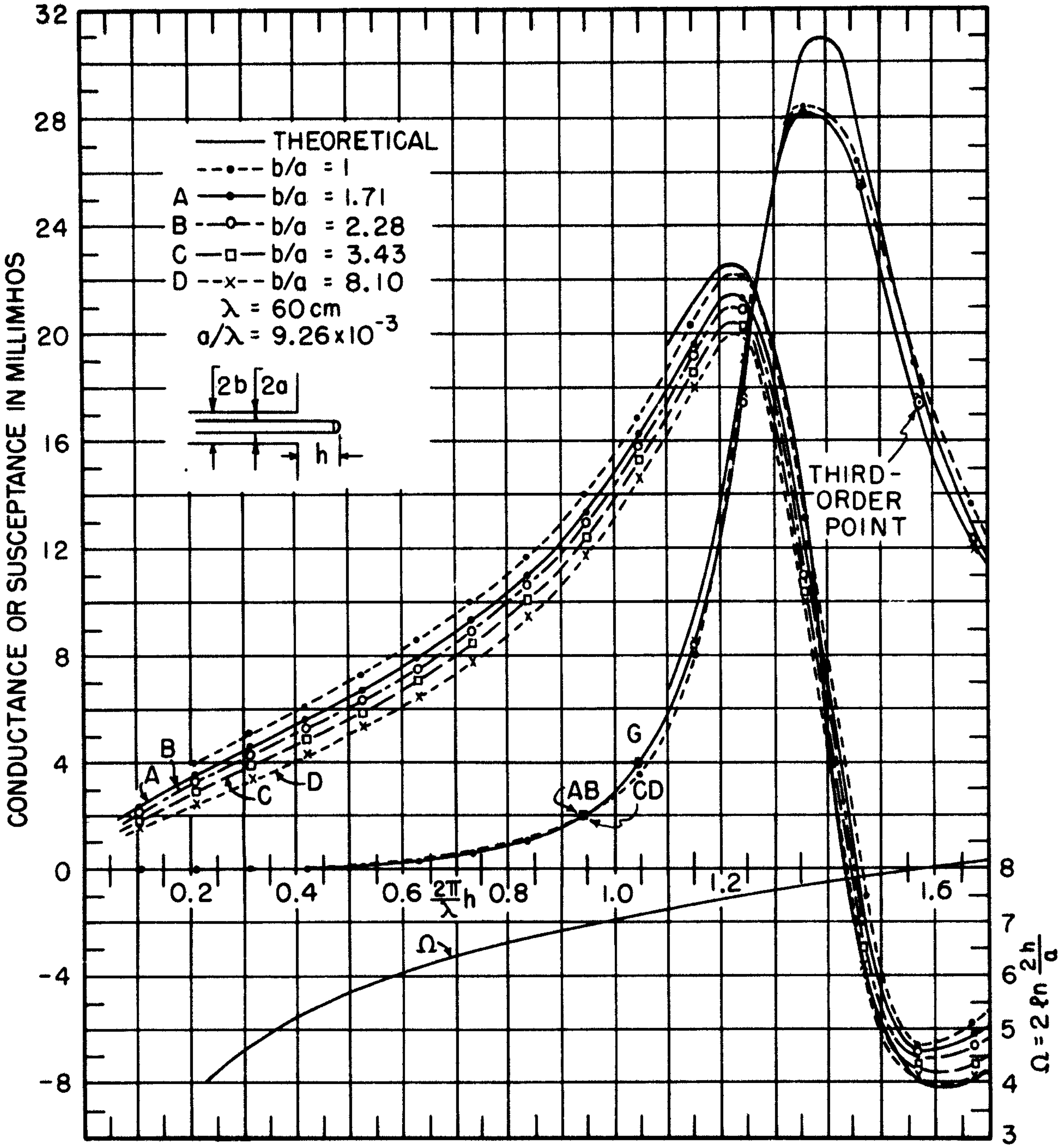
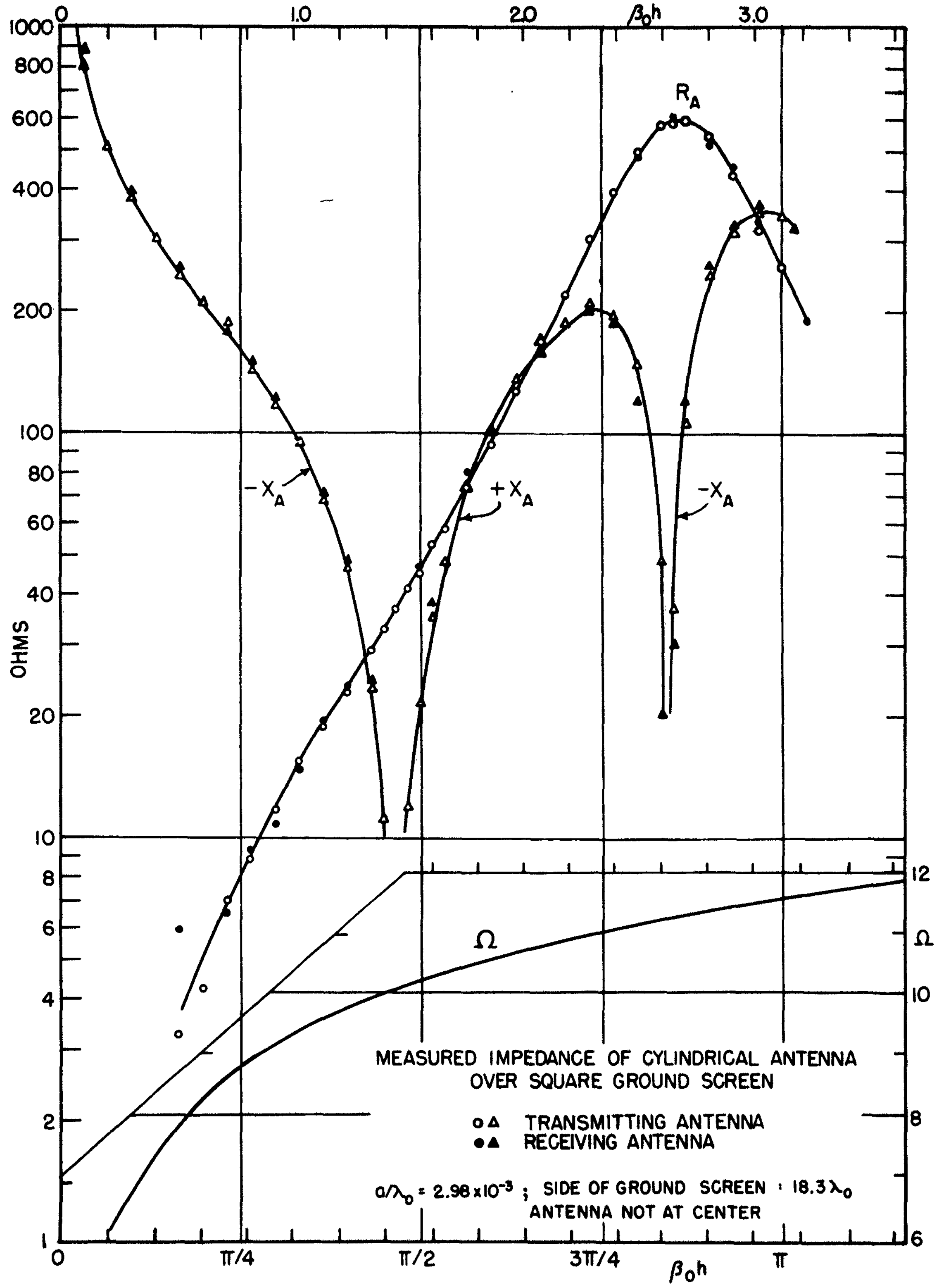
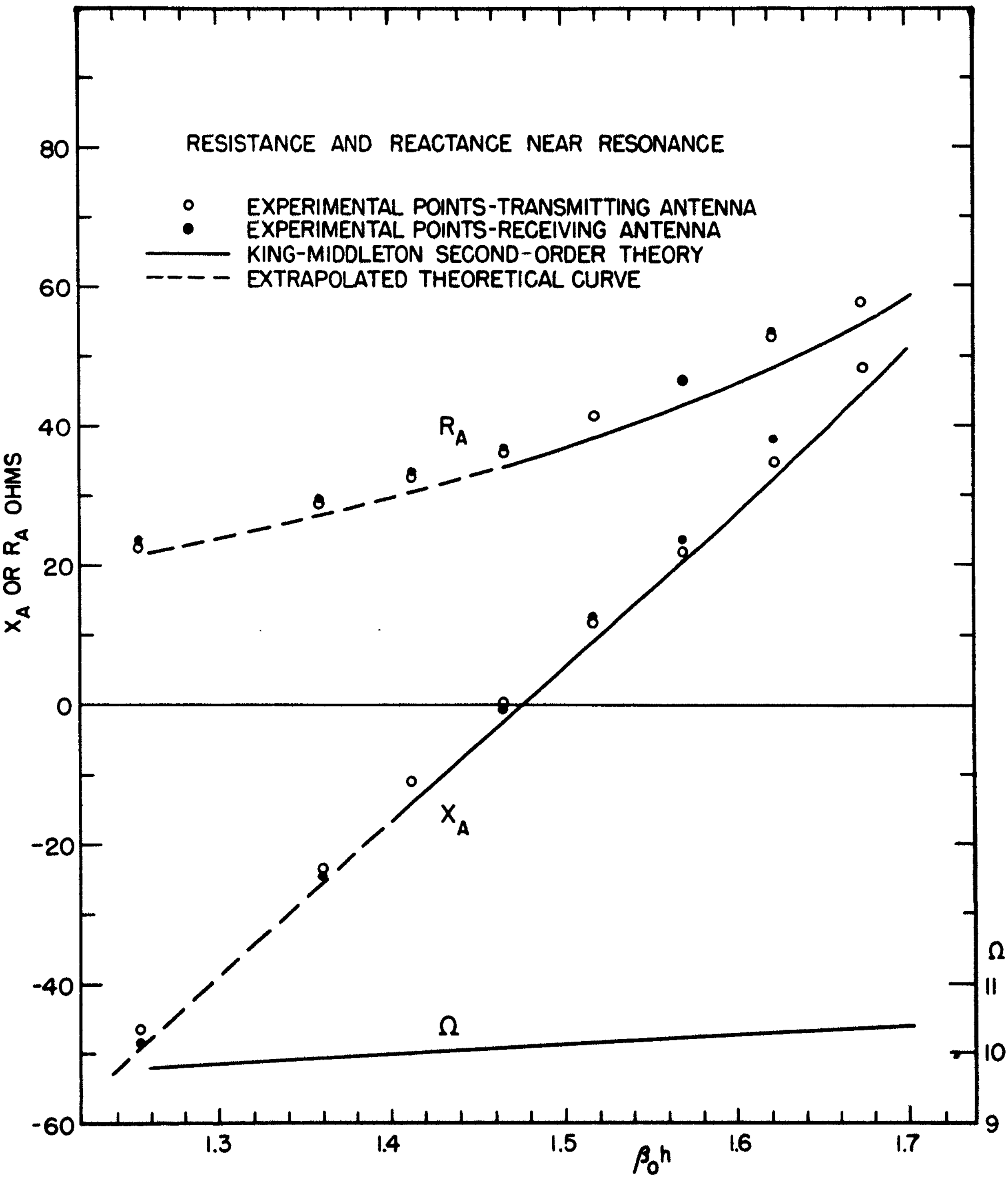


FIG. 33 ADMITTANCE NEAR RESONANCE FOR A MODERATELY THICK ANTENNA



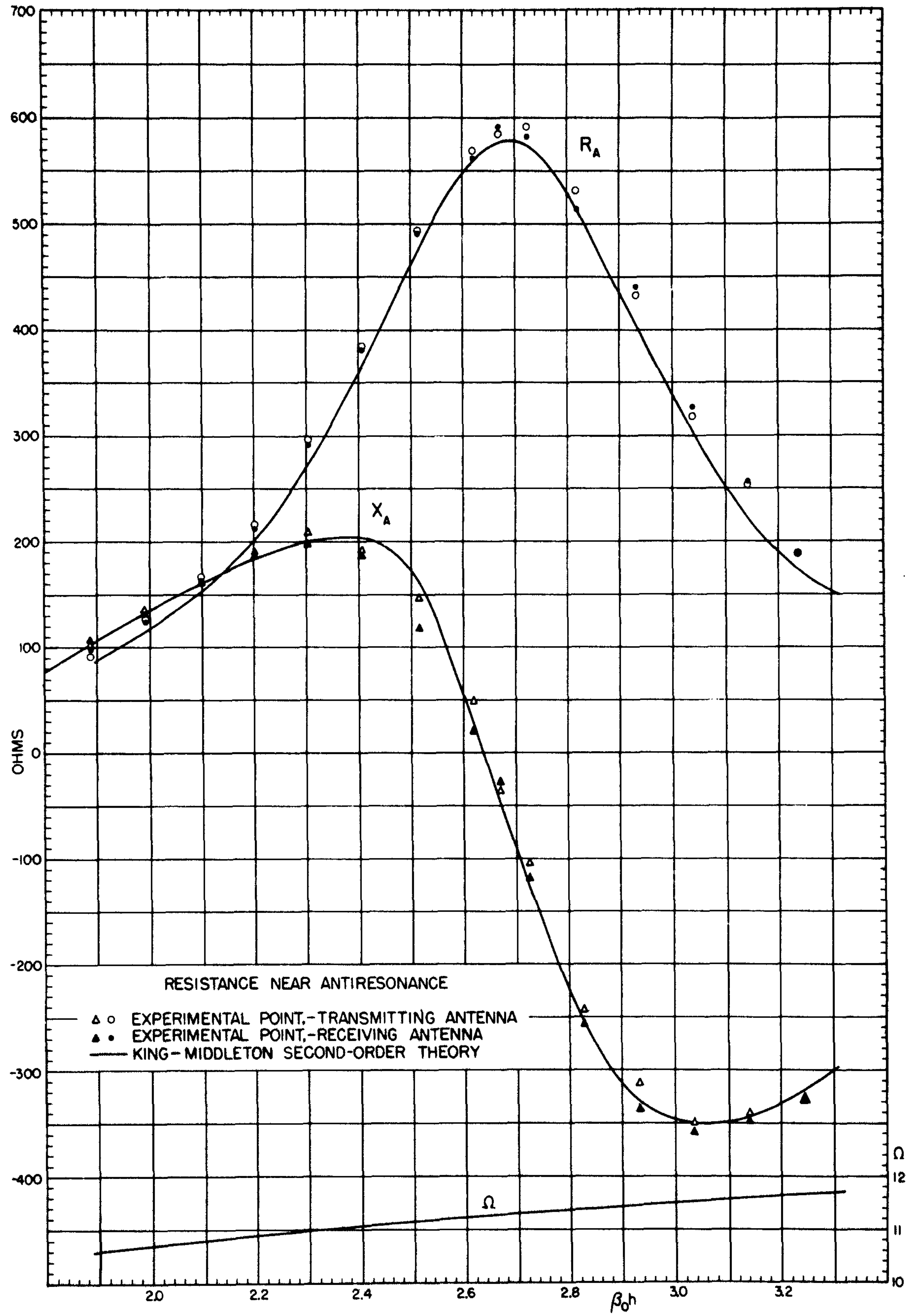
0
1.0
2.0
$\beta_0 h$
3.0
1000
800
600
400
200
100
80
60
40
20
10
8
6
4
2
1
OHMS
R_A
$-X_A$
$+X_A$
$-X_A$
Ω
12
Ω
10
8
6
0
$\pi/4$
$\pi/2$
$3\pi/4$
$\beta_0 h$
π
MEASURED IMPEDANCE OF CYLINDRICAL ANTENNA
OVER SQUARE GROUND SCREEN
○△ TRANSMITTING ANTENNA
●▲ RECEIVING ANTENNA
$a/\lambda_0 = 2.98 \times 10^{-3}$; SIDE OF GROUND SCREEN : $18.3\lambda_0$
ANTENNA NOT AT CENTER

FIGURE 34



RESISTANCE AND REACTANCE NEAR RESONANCE
EXPERIMENTAL POINTS-TRANSMITTING ANTENNA
EXPERIMENTAL POINTS-RECEIVING ANTENNA
KING-MIDDLETON SECOND-ORDER THEORY
EXTRAPOLATED THEORETICAL CURVE
X_A OR R_A OHMS
80
60
40
20
0
-20
-40
-60
R_A
X_A
Ω
Ω
11
10
9
1.3
1.4
1.5
1.6
1.7
$\beta_0 h$

FIGURE 35



700
600
500
400
300
200
100
0
-100
-200
-300
-400
OHMS
R_A
X_A
RESISTANCE NEAR ANTIRESONANCE
△ ○ EXPERIMENTAL POINT,-TRANSMITTING ANTENNA
▲ • EXPERIMENTAL POINT,-RECEIVING ANTENNA
KING-MIDDLETON SECOND-ORDER THEORY
Ω
12
11
10
2.0
2.2
2.4
2.6
2.8
3.0
3.2
$\beta_0 h$

FIGURE 36

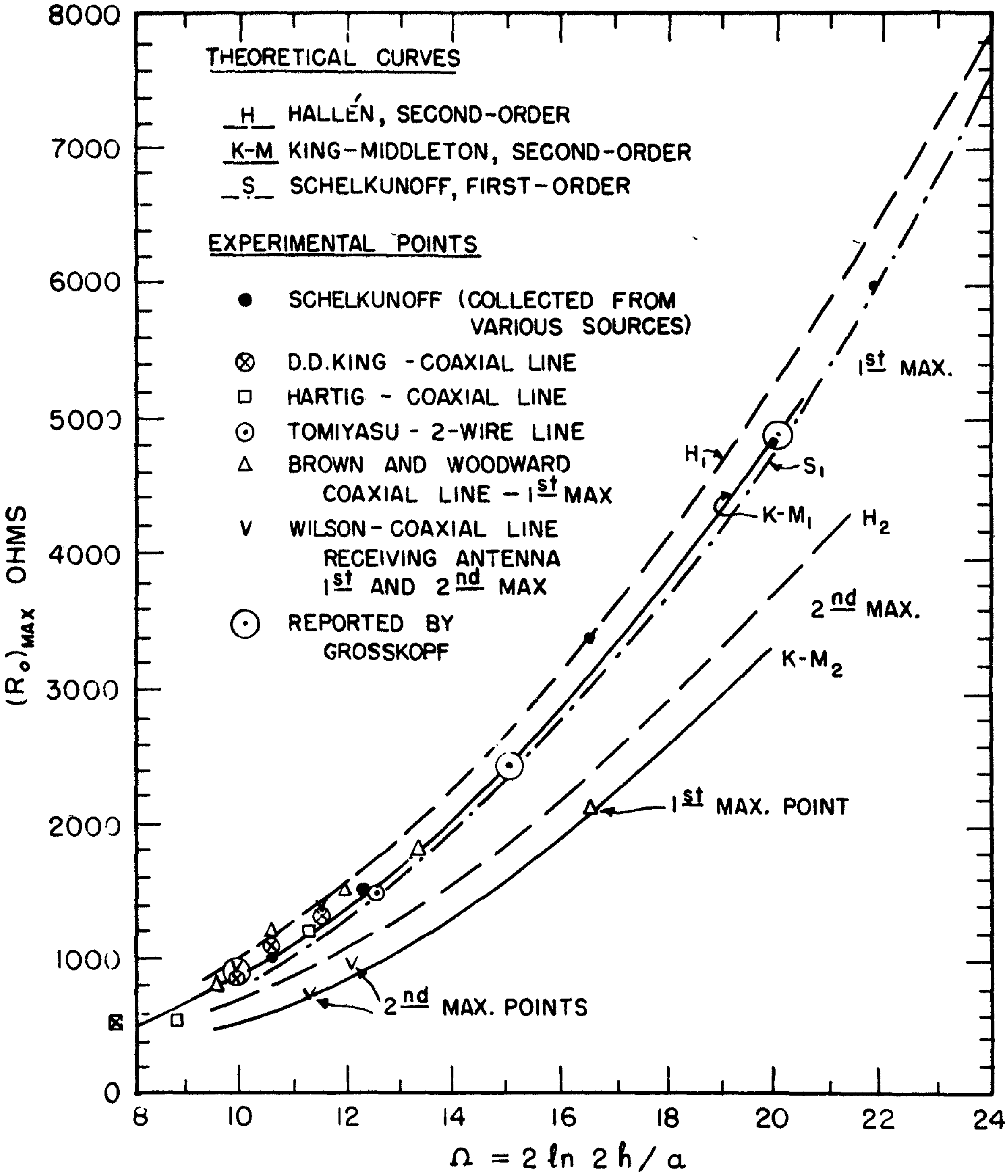


FIG. 37 RESISTANCE MAXIMA FOR CYLINDRICAL ANTENNA

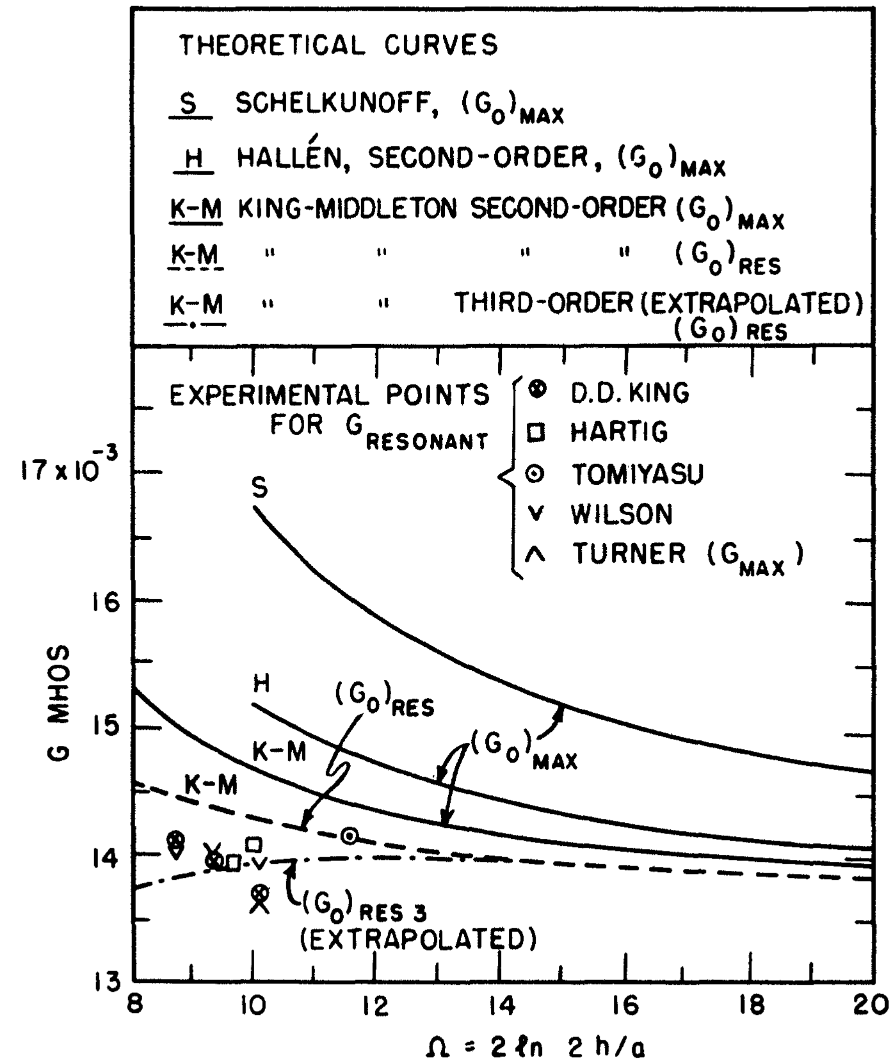


FIG. 38 CONDUCTANCE NEAR RESONANCE

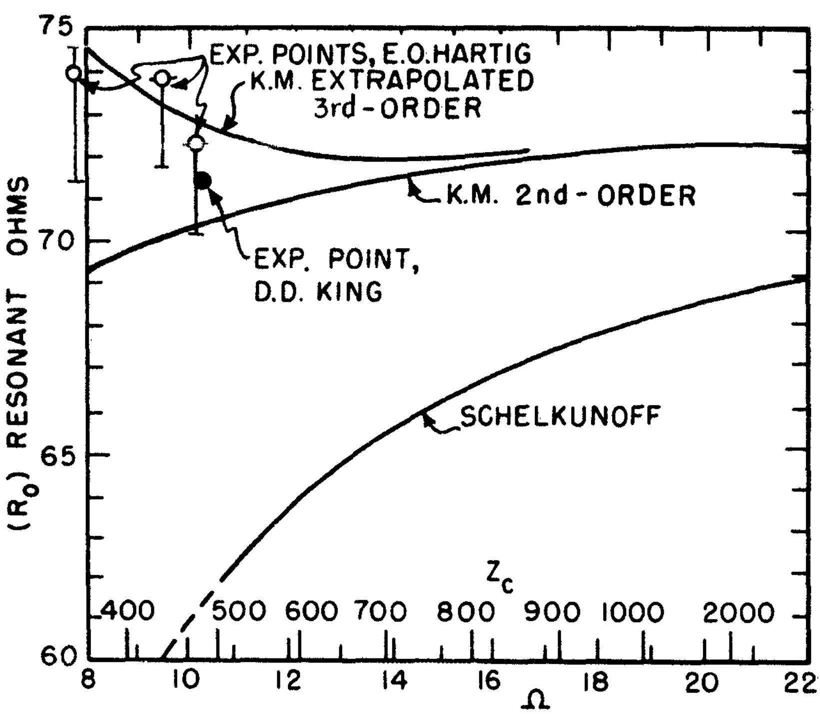


FIG. 39 RESISTANCE AT RESONANCE CYLINDRICAL ANTENNA

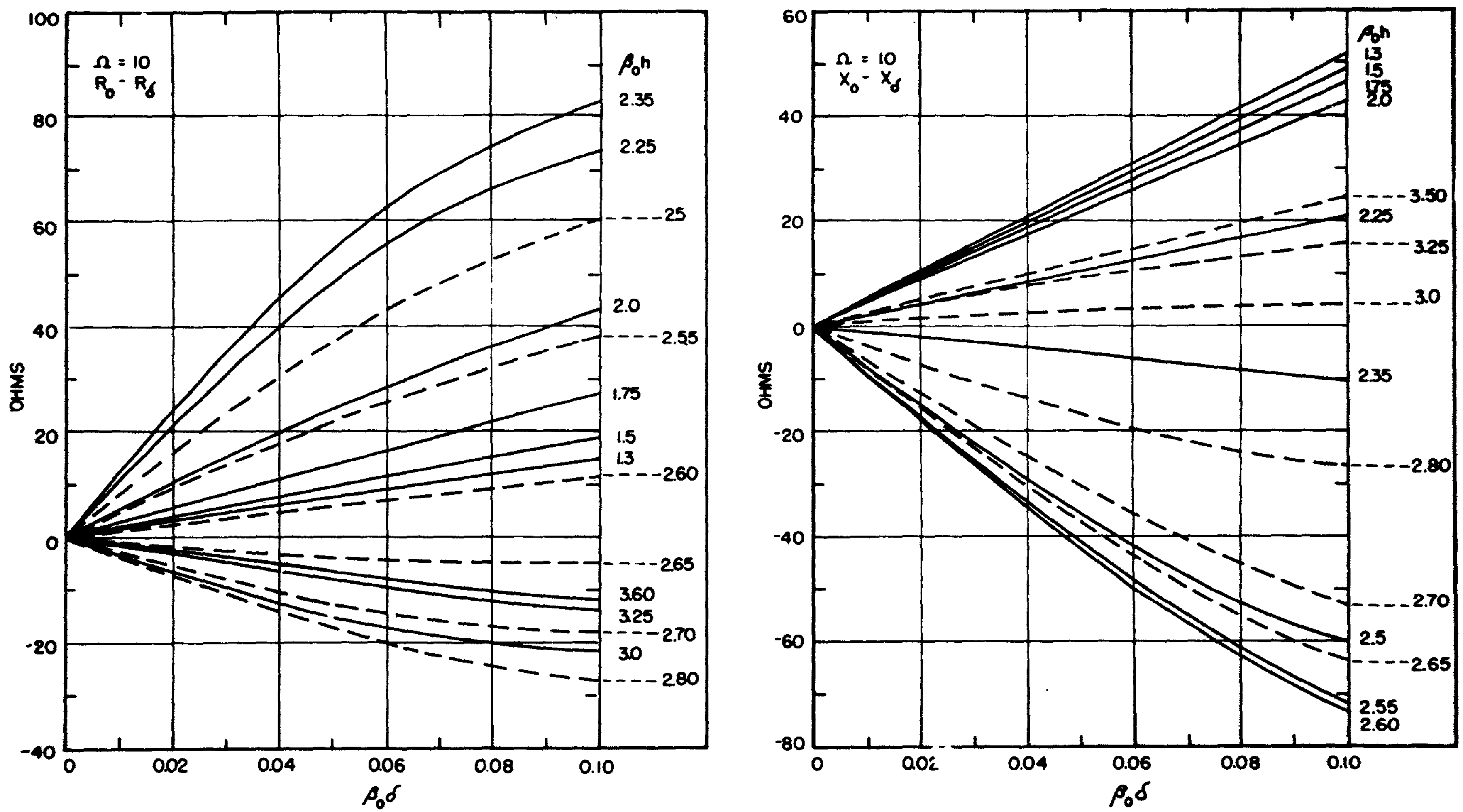


FIG. 40 LINE - SPACING CORRECTION FACTOR; $\Omega = 10$

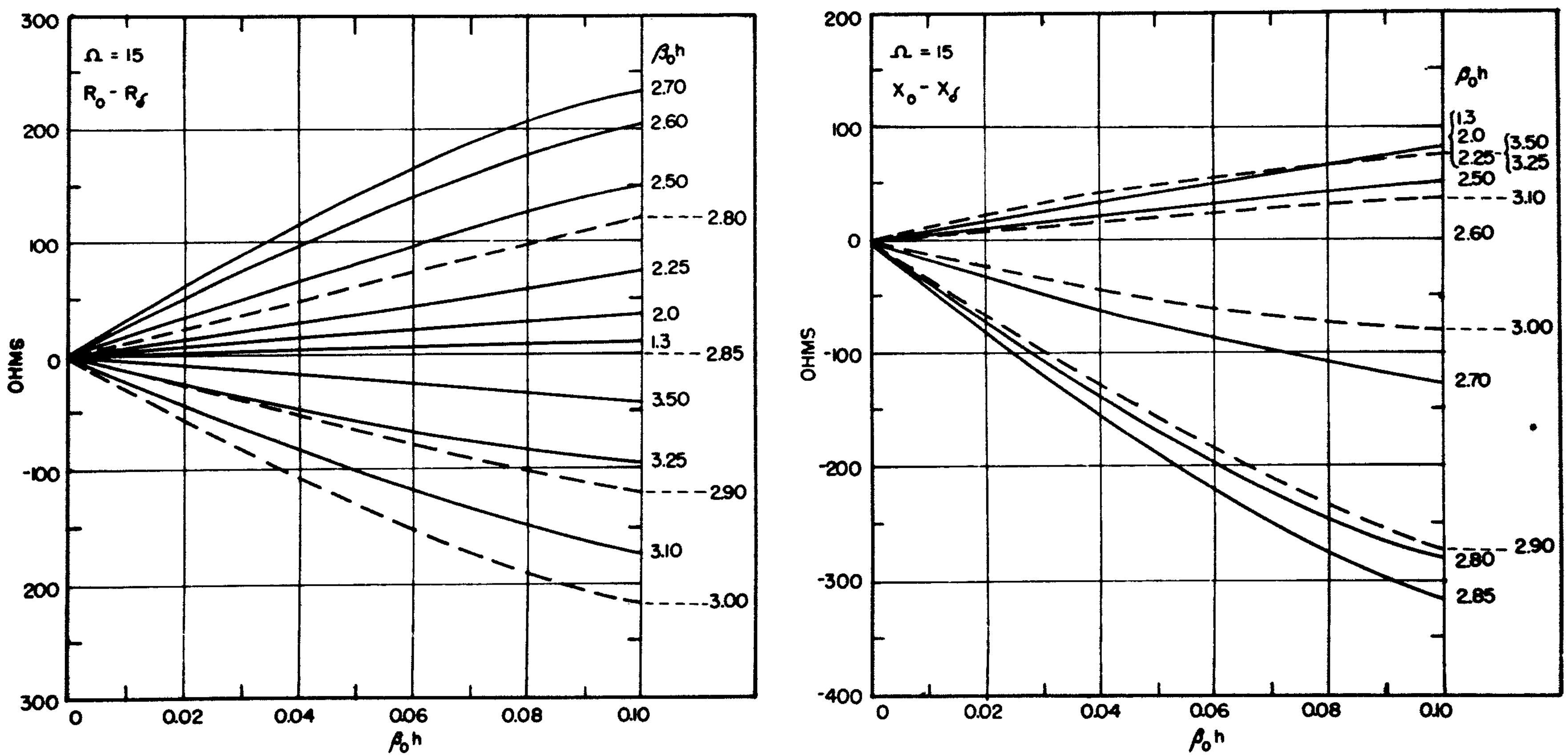


FIG. 41 LINE - SPACING CORRECTION FACTOR ; $\Omega = 15$

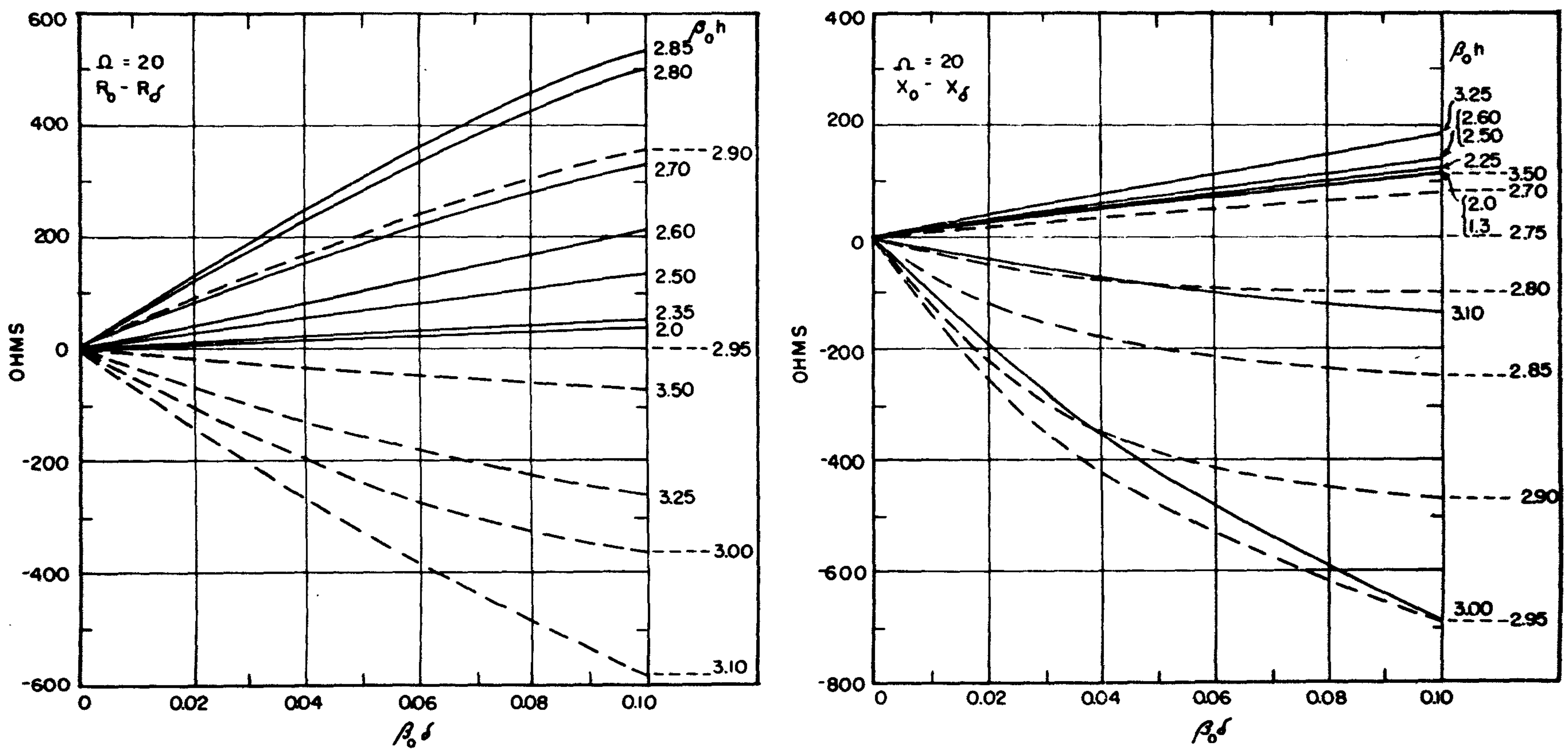


FIG. 42 LINE-SPACING CORRECTION FACTOR ; $\Omega = 20$